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Various Statistical Measures of a Parallel Redundant Complex System with Two Types of Failure

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Abstract

In this paper we analyze a Markov model consisting of two non-identical units in parallel redundancy. System suffers failure in two models; viz., normal mode (N.M) and common case (C.C). The repair, wherever undertaken follows general distribution. After the failure of one of the two units, the failure rate of another unit is assumed to be increased as compared to the situation on while it works jointly. This is termed as over loading effect.

Keywords: *Statistical Measures, Redundant Complex System, Types of Failure.*

1. Introduction

As the field of reliability engineering is becoming a recognized discipline in engineering, so is the awareness of its specialized topics which generally overlooked in the past. For example, recent years common cause failures have received a widespread attention in reliability analysis of redundant components, units or systems, because the assumption of statistical-independent failure of redundant units is easily violated in practice.

A common-cause failure is defined as any instance where multiple units or components fail due to a single cause.

In the recent past, several researchers[1,2,3,4,5] have contributed a lot in reliability field while analyzing various complex system mathematically, incorporating the concept of common-cause failure (may occur due to equipment design deficiency, operation and maintenance error, external environment, external catastrophe and function deficiency etc.).

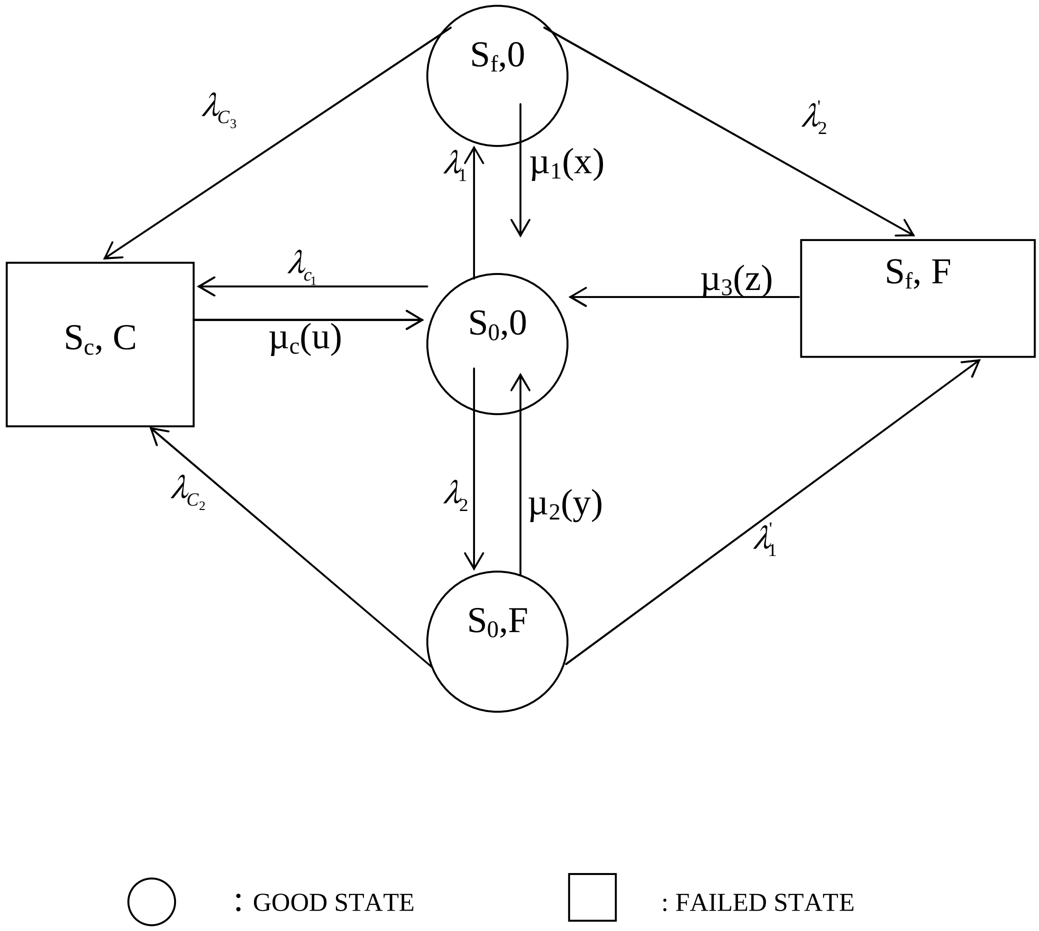
B.S. Dhillon and H.C. Viswanath [34] presented three models with common-cause failure and studied their reliability behavior. They considered a parallel redundant system consisting of two non-identical units assuming that the system or any single unit may collapse either due to normal failure or by the common-cause failure, they also assumed that repair and failure both follow exponential time distributions. The additional assumption in their analysis was that failure rate of either of the two units remains invariably the same whether it operates alone or jointly. The assumptions made by the earlier researchers are not realistic to practical situation problems as while one unit of the complex system fails. The failure rate of the other unit must increase positively due to the over-loading effect. Not only this , constant repair of the unit/system leads to wastage of time and cost both.

Keeping these facts in view, the author in this paper has therefore, analyzed a Markova's model consisting of two non-identical units in parallel redundancy. System suffers failure in two models; viz., normal mode (N.M) and common case (C.C). the repair, wherever undertaken follows general distribution. After the failure of one of the two units, the failure rate of another unit is assumed to be increased as compared to the situation on while it works jointly. This is termed as over loading effect.

Several reliability measures; viz., L.T. of various state probabilities, up and down state probabilities, M.T.T.F. (mean time to failure), variance, reliability of the system etc. have been computed using supplementary and L.T. techniques. To connect the utility of the model with practical life consideration, some graphs have also been appended at the end.

2. Assumptions

1. Initially, the system is good;
2. Units are non-identical and active;
3. A common-cause failure occurs at any of the operable states;
4. All failure rates are exponential whereas repair rates are arbitrary;
5. System is in failed state when both the units have failed;
6. Repair facility is available when either one unit or both the units have failed; and
7. After repair, system/unit works like a new.



STATE TRANSITION DIAGRAM

FIG.1

3. Notations

S_o o Good state of the system, when both the units are active;
S_f, o Good state of the system, when first unit has failed and second is active;
S_o f Good state of the system, when second units has failed and first is active;
S_f, f Failed state of the system when both the units have failed due to N.M;
S_c, c Failed state of the system due to C.C;
λ_1 / λ_2 Failure rate of first/second unit when operating jointly;
λ_1' / λ_2' Failure rate of first/second unit when operating alone;
λ C_1 Common-cause failure rate of the system when both units are good;
λ C_2/ λ C_3 Common-cause failure rate of the system when first/second unit is active;
$\frac{\mu_1(x)}{\mu_2(y)}$ Transition repair rate from state S_f, o/
$\frac{\mu_3(z)}{\mu_c(u)}$ $S_{o,f}$ / $S_{f,f}$ / $S_{c,c}$ to $S_{o,o}$;
$P_{i,j}(x,t) / P_{i,jy,t)}$ / General pdf [system is in state $S_{i,j}$ and
$P_{i,j}(z,t)$ / $P_{i,j}$ (u,t) is under repair; elapsed repair time is x/y/z/u;t]
[(i.j) means (f,o), (o,f) (f,f), (c,c)];
$P_{i,j}$ (t) Pr[system is in state S_i, j ; t];

$\bar{f}(s)$ Laplace-transform of the function f (t);
S Laplace-transform variable;
$\int$ Integrated notation (o,∞), unless otherwise stated;
S_i (x) $\mu_i(x)\ \exp\left[-\int_o^x \mu i(x)dx\right]$;
Mi Mean time to repair.

4. Formulation of Mathematical Model

Elementary probability consideration and continuity arguments yield the following difference-differential equations associated with fig.1

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+\lambda_{C1}\right]P_o,o(t)=\int_o^\infty P_f,o(x,t)\mu_1(x)dx+\int_o^\infty P_0,f(y,t)\mu_2(y)dy+$$
$$\int_o^\infty P_f,f(z,t)\mu_3(z)dz+\int_o^\infty P_c,c(u,t)\mu_c(u)du \tag{1}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\lambda_2^1+\lambda_{c_3}+\mu_1(x)\right]P_f,o(t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda_1^1+\lambda_{c_2}+\mu_2(y)\right]P_o,f(t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_3(z)\right]P_f,f(t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial u}+\frac{\partial}{\partial t}+\mu_c(u)\right]P_c,c(t)=0 \quad (5)$$

Boundary Conditions

$$P_f,0(0,t)=\lambda_1 P0,0(t) \quad (6)$$

$$P_0,f(0,t)=\lambda_2 P0,0(t) \quad (7)$$

$$P_f,f(0,t)=\lambda_2^1 P_f,0(t)+\lambda_1^1 P_0,f(t) \quad (8)$$

$$P_c,c(0,t)=\lambda_{c_1}P_0,0(t)+\lambda_{c_3}P_f,0(t)+\lambda_{c_2}P_0,f(t) \quad (9)$$

Initial Conditions

$$\left[\begin{array}{l}\mathrm{P_o},0(0)=1\\ \mathrm{P_f},0(0)=\mathrm{P_o},\mathrm{f}(0)=\mathrm{P_f},\mathrm{f}(0)=\mathrm{P_c},\mathrm{c}(0)=0\end{array}\right. \quad (10)$$

5. Solution of the Model

Taking Laplace-transforms of equations (1-9) and using equation (10), one may obtain:-

$$[S+\lambda_1+\lambda_2+\lambda_{c1}]\bar{P}_o,o(s)=1+\int_o^\infty P_f,o(x,s)\mu_1(x)dx+\int_o^\infty P_o,f(y,s)\mu_2(y)dy+ \int_o^\infty P_f,f(z,s)\mu_3(z)dz+\int_o^\infty P_c,c(u,s)\mu_c(u)du \quad (11)$$

$$\left[\frac{\partial}{\partial x}+s+\lambda_2^1+\lambda_{c_3}+\mu_1(x)\right]\bar{P}_f,o(x,s)=0 \quad (12)$$

$$\left[\frac{\partial}{\partial y}+s+\lambda_1^1+\lambda_{c_2}+\mu_2(y)\right]\bar{P}_0,f(y,s)=0 \quad (13)$$

$$\left[\frac{\partial}{\partial z}+s+\mu_3(z)\right]\bar{P}_f,f(z,s)=0 \quad (14)$$

$$\left[\frac{\partial}{\partial u}+s+\mu_c(u)\right]\bar{P}_c,c(u,s)=0 \quad (15)$$

$$\bar{P}_f,0(0,s)=\lambda_1\bar{P}_0,0(s) \quad (16)$$

$$\bar{P}_0,f(0,s)=\lambda_2\bar{P}_0,0(s) \quad (17)$$

$$\bar{P}_f,f(0,s)=\lambda_2^1\bar{P}_f,0(s)+\lambda_1^{'}\bar{P}_0,f(s) \quad (18)$$

$$\bar{P}_c,c(0,s)=\lambda_{c_1}\bar{P}_0,0(s)+\lambda_{c_3}\bar{P}_f,0(s)+\lambda_{c_2}\bar{P}_0,f(s) \quad (19)$$

6. Calculation of $\bar{P}_i(S)$

In view of equation (16-19), equations (11-15) yield the following Laplace-transforms of the system's state probabilities:-

$$\bar{P}_0,0(s) = \frac{1}{A(s)} \tag{20}$$

$$\bar{P}_f,0(s) = \lambda_1 z_1 (s + \lambda_2^1 + \lambda_{c_3}) \frac{1}{A(s)} \tag{21}$$

$$\bar{P}_0, f(s) = \lambda_2 z_2 (s + \lambda_1^1 + \lambda_{c_2}) \frac{1}{A(s)} \tag{22}$$

$$\bar{P}_f, f(s) = \left[\lambda_2^1 \lambda_1 z_1 (s + \lambda_2^1 + \lambda_{c_3}) + \lambda_1^{'} \lambda_2 z_2 (s + \lambda_1^{'} + \lambda_{c_2})\right] x \frac{Z_3(s)}{A(s)} \tag{23}$$

$$\bar{P}_c, c(s) = \left[\lambda_{c_1} + \lambda_{c_3} \lambda_1 z_1 (s + \lambda_2^1 + \lambda_{c_3}) + \lambda_{c_2}^{'} \lambda_2 z_2 (s + \lambda_1^{'} + \lambda_{c_2})\right] \frac{z_c(s)}{A(s)} \tag{24}$$

Also, L.T. of the probability that the system is in operable state at time t is given by

$$\bar{P}_{up}(s) = \left[1 + \lambda_1 z_1 (s + \lambda_2^1 + \lambda_{c3}) + \lambda_2 z_2 (s + \lambda_1^{'} + \lambda_{c2})\right] \times \frac{1}{A(s)} \tag{25}$$

Further, the L.T. of the probability that the system is in failed state at time t is given by

$$\bar{P}_{down(s)} = \left[\lambda_{c_1} Z_c(s) + \{\lambda_2^1 Z_3(s)\} + \lambda_{C_3} Z_c(s)\right] \times \lambda_1 Z_1 (s + \lambda_2^1 + \lambda_{c_3}) + \{\lambda_1^{'} Z_3(s) + \lambda_{c_2} Z_c(s)\} \times$$
$$\lambda_2 Z_2 (s + \lambda_1^{'} + \lambda_{c_2})] \frac{1}{A(s)} \tag{26}$$

Where

$$Z_i(s) = \frac{1 - \bar{S}_i(s)}{S}$$

$$A(s) = \begin{bmatrix} S + \lambda_1 + \lambda_2 + \lambda_{c_1} - \lambda_1 \bar{S}_1 (s + \lambda_2^1 + \lambda_{c_3}) - \lambda_2 \bar{S}_2 (s + \lambda_1^{'} \lambda_{c_2}) - \{\lambda_2^{'} \lambda_1 Z_1 (s + \lambda_2^{'} \lambda_{c_3}) + \\ \lambda_1^{'} \lambda_2 Z_2 (s + \lambda_1^{'} \lambda_{c_2})\} \bar{S}_3(s) - \{\lambda_{c1} + \lambda_{c_3} \lambda_1 Z_1 (s + \lambda_2^{'} + \lambda_{c_3}) + \lambda_{c_2} \lambda_2 Z_2 (s + \lambda_1^{'} + \lambda_{c_2})\} \bar{S}_c(s) \end{bmatrix}$$

It is worth noticing that

$$\bar{P}_{up}(s) + \bar{P}_{down}(s) = \frac{1}{3} \tag{27}$$

7. Steady State Behavior of the System

Using Abel's Lemma; viz. $\lim_{t \to \infty} P(t) = \lim_{S \to 0} S \bar{P}(S)$

P (say), we get the following steady state probabilities of the system:-

$$P_0, 0 = \frac{1}{A'(0)} \tag{28}$$

Where

A'(0) = $\frac{d}{ds}[A(s)]s = 0 \;\; P_f, 0 = \lambda_1 z_1(\lambda_2^1 + \lambda_{c_3}) \frac{1}{A(0)}$ (29)

$$P_0, f = \lambda_2 z_2(\lambda_1^{'} + \lambda_{c_2}) \frac{1}{A'(0)} \tag{30}$$

$$P_f, f = \left[\lambda_2^1 \lambda_1 Z_1(\lambda_2^1 + \lambda_{c_3}) + \lambda_1^{'} \lambda_2 Z_2(\lambda_1^{'} + \lambda_{c_2})\right] \times \frac{M_c}{A'(0)} \tag{31}$$

$$P_c, c = \left[\lambda_{c_1} \lambda_{c_3} \lambda_1 Z_1(\lambda_2^1 + \lambda_{c_3}) + \lambda_{c_2}^{'} \lambda_2 Z_2(\lambda_1^{'} + \lambda_{c_2})\right] \times \frac{M_c}{A'(0)} \tag{32}$$

Also up and down state probabilities of the system are given by

$$P_{up} = \left[1 + z_1(\lambda_2^{'} + \lambda_{c3}) + \lambda_2 z_2(\lambda_1^{'} + \lambda_{c_2})\right] \frac{1}{A'(0)} \tag{33}$$

$$P_{down} = \begin{bmatrix} \{\lambda_2^{'} \lambda_1 Z_1(\lambda_2^{'} + \lambda_{c_3}) + \lambda_1^{'} \lambda_2 Z_2(\lambda_1^{'} + \lambda_{c_2})\} M_c \\ + \{\{\lambda_{c_1} + \lambda_{c_3} \lambda_1 Z_1(\lambda_2^{'} + \lambda_{c_3}) + \lambda_{c_2} \lambda_2 Z_2(\lambda_1^{'} + \lambda_{c_2})\} MC \end{bmatrix} \times \frac{1}{A'(0)} \tag{34}$$

8. Particular Case

When repair follows exponential time distribution

Setting, $\bar{S}_i(s) = \frac{\mu i}{s + \mu i} (i = 1,2,3,c)$ in equations (25) and (26), one may easily obtain:-

$$\bar{P}_{up}(s) = \left[1 + \frac{\lambda_1}{s + \lambda_2^{'} + \lambda_{c3} + \mu_1} + \frac{\lambda_1}{s + \lambda_1^{'} + \lambda_{c2} + \mu_2}\right] \times \frac{1}{A_1(s)} \tag{35}$$

$$\bar{P}_{down}(s) = \begin{bmatrix} \frac{\lambda_{c_1}}{s + \mu_c} + \left\{\frac{\lambda_2^{'}}{s + \mu_3} + \frac{\lambda_{c_3}}{s + \mu_c}\right\} \frac{\lambda_1}{s + \lambda_2^{'} + \lambda_{c_3} + \mu_1} \\ = \left\{\frac{\lambda_1^{'}}{s + \mu_3} + \frac{\lambda_{c2}}{s + \mu_c}\right\} \frac{\lambda_2}{s + \lambda_1^{'} + \lambda_{c_2} + \mu_2} \end{bmatrix} \times \frac{1}{A_1(s)} \tag{36}$$

Where

$$A_1(s) = s + \lambda_1 + \lambda_2 + \lambda_{c1} - \frac{\lambda_1 \mu_1}{s + \lambda_2^{'} + \lambda_{c_3} + \mu_1} - \frac{\lambda_2 \mu_2}{s + \lambda_1^{'} + \lambda_{c_2} + \mu_2}$$

$$-\left[\frac{\lambda_2^{'} \lambda_1}{s + \lambda_2^{'} + \lambda_{c_3} + \mu_1} + \frac{\lambda_1^{'} \lambda_2}{s + \lambda_1^{'} + \lambda_{c_2} + \mu_2}\right] \frac{\mu_3}{s + \mu_3}$$

$$-\left[\lambda_{c1} + \frac{\lambda_{c_3} \lambda_1}{s + \lambda_2^{'} + \lambda_{c_3} + \mu_1} + \frac{\lambda_{c_2} \lambda_2}{s + \lambda_1^{'} + \lambda_{c_2} + \mu_2}\right] \frac{\mu_C}{s + \mu_c}$$

Laplace-transform of the reliability of the system is then given by

$$\bar{R}(s) = \frac{1}{s + \lambda_1 + \lambda_2 + \lambda_{c_1}} + \frac{\lambda_1}{(s + \lambda_2^{'} + \lambda_{c_3})(s + \lambda_1 + \lambda_2 + \lambda_{c_1})} + \frac{\lambda_2}{(s + \lambda_1^{'} + \lambda_{c_2})(s + \lambda_1 + \lambda_2 + \lambda_{c_1})} \tag{37}$$

Meantime to failure (M.T.T.F) of the system is given by

$$\text{M.T.T.F} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_{c_1}} \left[1 + \frac{\lambda_1}{\lambda_2^{'} + \lambda_{c_3}} + \frac{\lambda_2}{\lambda_1^{'} + \lambda_{c_2}}\right] \tag{38}$$

Also, variance of the time to failure is given by

$$\sigma^2 = \left[\lambda_2^{'} + \lambda_{c_3}\right]^2 \left(\lambda_1^{'} + \lambda_{c_2}\right)^2 - \left\{\left(\lambda_1^{'} + \lambda_{c_2}\right)\lambda_1 + \left(\lambda_2^{'} + \lambda_{c_3}\right)\lambda_2\right\}^2$$
$$+ 2\left\{\left(\lambda_1^{'} + \lambda_{c_2}\right)^2 . \lambda_1 + \left(\lambda_2^1 + \lambda_{c_3}\right)^2 \lambda_2\right\}\left(\lambda_1 + \lambda_2 + \lambda_{c_1}\right)\Big] \times \left[\left(\lambda_1 + \lambda_2 + \lambda_{c_1}\right)\left(\lambda_2^{'} + \lambda_{c_3}\right)\left(\lambda_1^{'} + \lambda_{c_2}\right)\right]^{-2} \tag{39}$$

After taking inverse L.T., equation (37) yields the reliability of the system at time t; viz.

$$R(t) = \left[1 + \lambda_1\left(\lambda_2^1 + \lambda_{c_3} - \lambda_1 + \lambda_2 - \lambda_{c_1}\right)^{-1} + \lambda_2\left(\lambda_1^{'} + \lambda_{c_2} - \lambda_1 - \lambda_2 - \lambda_{c_1}\right)^{-1}\right] \times$$
$$\exp.\left[-\left(\lambda_1 + \lambda_2 + \lambda_{c_1}\right)t\right] + \lambda_1\left(\lambda_1 + \lambda_2 + \lambda_{c_1} + \lambda_2^{'} - \lambda_{c_3}\right)^{-1}$$
$$\exp.\left[-\left(\lambda_2^{'} + \lambda_{c_3}\right)t\right] + \lambda_2\left(\lambda_1 + \lambda_2 + \lambda_{c_1} - \lambda_1^{'} - \lambda_{c_2}\right)^{-1}$$
$$\exp.\left[-\left(\lambda_1^{'} + \lambda_{c_2}\right)t\right] \tag{40}$$

9. Numerical Computation

Setting some suitable value of $\lambda_1, \lambda_2, \lambda_2^{'}, \lambda_2^{'}, \lambda_{c2}$

λ_{c_3} in equation (38) and of $\lambda_1, \lambda_2, \lambda_{c1}, \lambda_1, \lambda_2, \lambda_{c2}$ and λ_{c_3} in equation (40),one may sketch the graphs; M.T.T.F.

V.$N_{c1}$and reliablitiy V.time respectively, which are shown in the adjoining figures.

Table 1

$\lambda_1 = 0.01, \lambda_2 = \lambda_1^{'} = 0.02, \lambda_2^{'} = 0.03$

Table 1

S.1 No.	λ_{c_1}	M.T.T.F	
		$\lambda_{c_2} = \lambda_{c_3}$ =0	$\lambda_{c_2} = \lambda_{c_3}$ =0.0005
1	0.002	72.91	71.98
2	0.004	68.62	67.74
3	0.006	64.81	63.98
4	0.008	61.40	60.61
5	0.010	58.33	57.58
6	0.012	55.55	54.84
7	0.014	53.03	52.35
8	0.016	50.72	50.07
9	0.018	48.61	47.98
10	0.020	46.66	46.06

Table 2

$\lambda_1 = 0.01, \lambda_2 = \lambda_1^{'} = 0.02, \lambda_2^{'} = 0.03$

S.$_1$ No.	Time (t)	R(t)	
		$\lambda_{c_1} = \lambda_{c_2} = \lambda_{c_3} = 0$	$\lambda_{c_1} = 0.005, \lambda_{c_2} = 0.06, \lambda_{c_3} = 0.007$
1	0	1	1
2	1	0.9995	0.9946
3	2	0.9985	0.9886
4	3	0.9969	0.9820
5	4	0.9947	0.9748
6	5	0.9919	0.9670
7	6	0.9887	0.9589
8	7	0.9848	0.9501
9	8	0.9805	0.9411
10	9	0.9757	0.9316
11	10	0.9706	0.9219

M.T.T.F/ λ_{C_1}

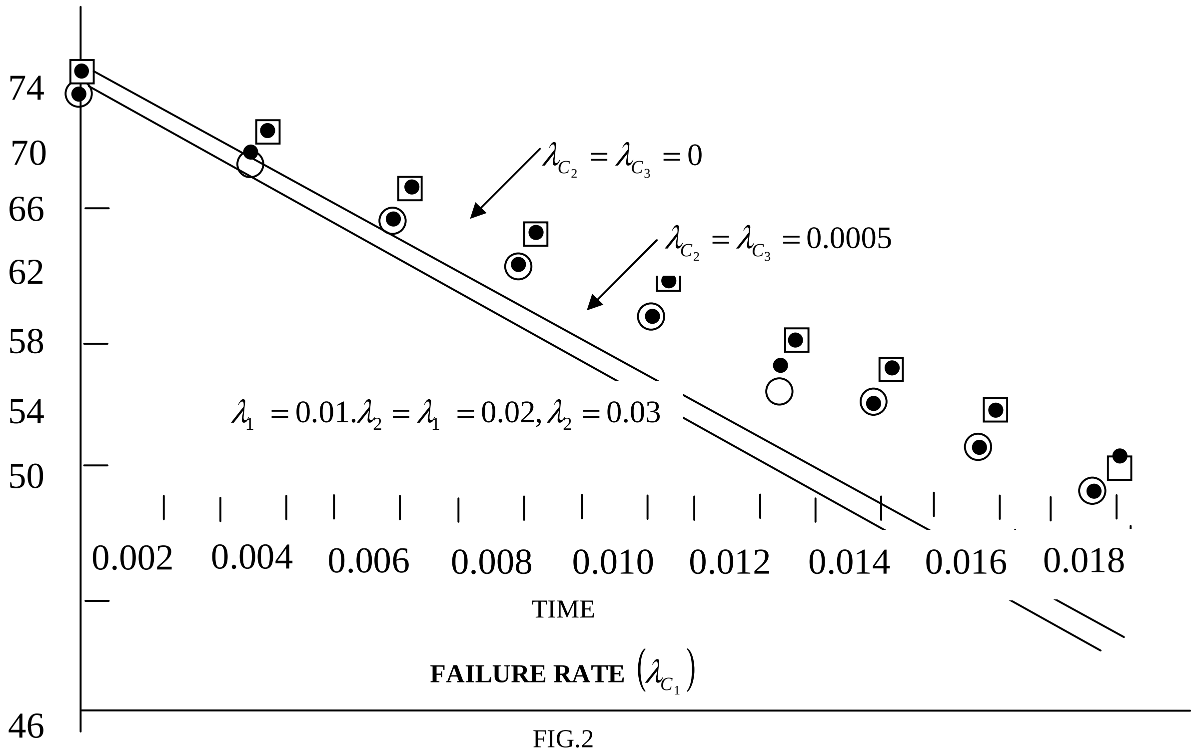


FIG.2

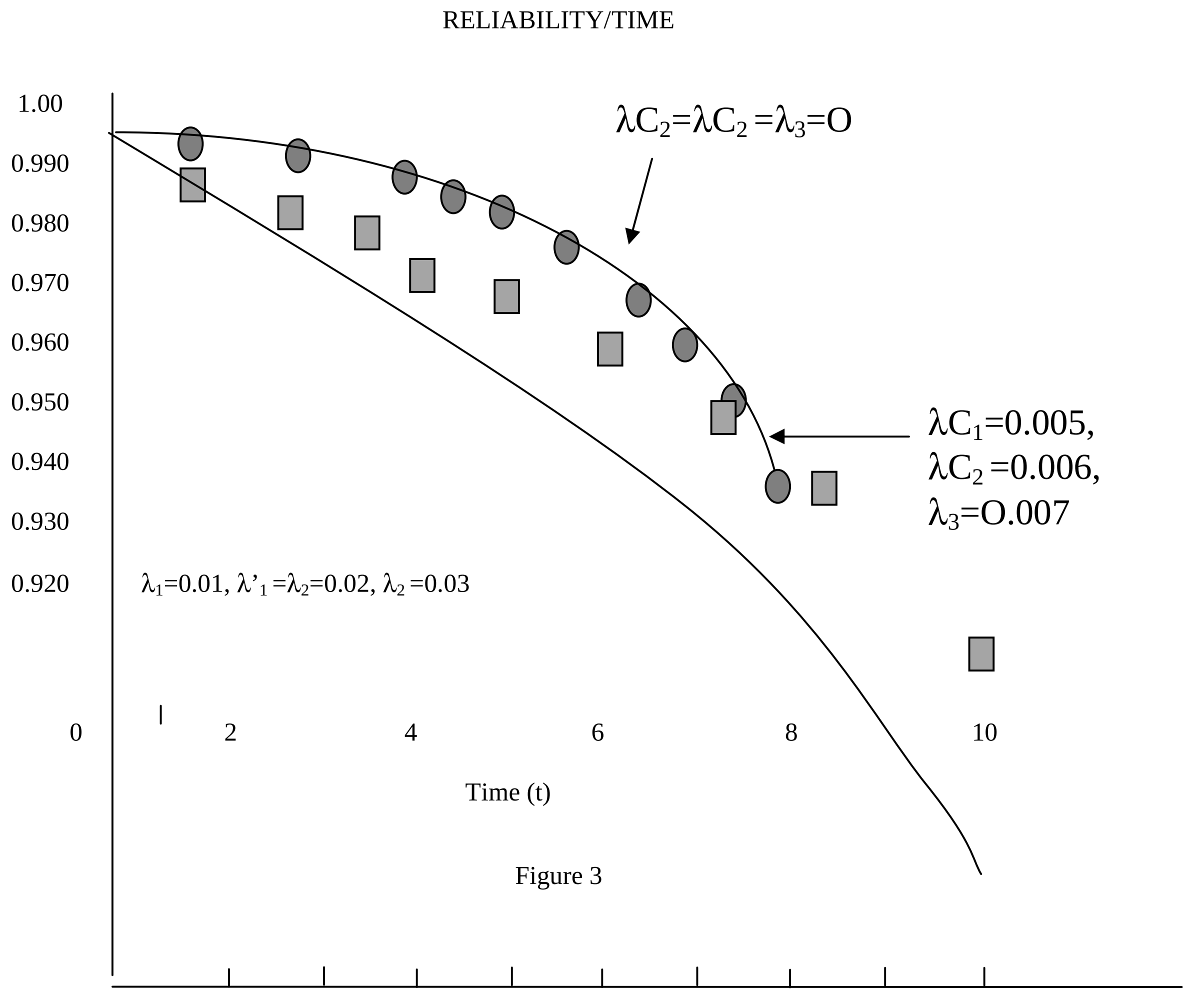


Figure 3

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