

# Empirical Comparison & Analysis of Shuffle Exchange Network and Its Variants

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## Abstract

Shuffle-exchange networks (SENs) have been widely considered as practical interconnection systems due to their size of its switching elements (SEs) and uncomplicated configuration. SEN is a network among a large class of topologically equivalent multistage interconnection networks (MINs) that includes omega, indirect binary n-cube, baseline, and generalized cube. In this paper, SEN with additional stages that provide more redundant paths are analyzed. A common network topology with a  $2 \times 2$  basic building block in a SEN and its variants in terms of extra-stages is investigated. Finally terminal reliability of SEN, SEN+, SEN+2 are analyzed.

**Keywords:** *Interconnection; Multistage; Networks; Reliability; Fault tolerant; Shuffle exchange; Stages; Switching elements.*

## I. INTRODUCTION

Interconnection networks play a major role in the performance of modern parallel computers. These networks can provide the communication in a parallel processing system consisting of a large number of processors that are working together to perform a single overall task. For large multiprocessor systems, the shared bus and the crossbar switch represent the extremes in interconnection schemes. The shared bus is inexpensive, but it is too slow and has limited bandwidth when a large number of processors must rely on it for communication. At the other extreme, the crossbar switch provides high bandwidth and fastest possible communication speed, but its cost grows with the square of number of processors.

Multistage interconnection networks (MIN) provide a compromise between the shared bus and the crossbar

switch. These networks are designed to provide fast and efficient communication at a reasonable cost.

MINs consist of layers of switching elements (SEs) with a specific topological pattern. These networks provide interconnection between the set of processors (inputs) and the set of memory modules (outputs). They fall within the category of indirect network as they rely on intermediate elements to provide the interconnection between the input and output elements. It has been extensively used in both circuit switching and packet switching networks with the introduction of buffered switch. These include multiprocessor and communication network environments such as Ultra computer, IBM RP3,

ATM switches, and optical network. The number of stages, interconnection topology, and the type of SEs used in the network configuration differentiate each MIN fault tolerant. Examples of the widely used MINs include: shuffle Exchange network (SEN), Gamma network, extra-stage Gamma network, delta network, Tandem – Banyan network.

Due to the size of its SE and uncomplicated configuration of SEN as shown in Fig.1, it is one of the most commonly used MINs. Regular multi-stage interconnection networks have an equal number of switching elements per stage; as a result they may impose equal time delay to all the requests passing through them.

In this paper, MINs with additional stages that provide more redundant paths are analyzed. A common network topology with a  $2 \times 2$  basic building block such as SEN and its variants in terms of extra-stages is presented first. Three types of SENs are studied: SEN, SEN with an additional stage (SEN+), and SEN with two additional stages (SEN+2). Three measures of network system reliability: terminal, broadcast, and network reliability are developed to evaluate the performance of these networks and to

assess the effect of additional stages on SEN reliability.

**II. DESCRIPTION OF SEN, SEN+, SEN+2**

While we have chosen to analyze the SEN, this work can be extended to many other MINs since the SEN is just one network in large class of topological equivalent MINs that include the Omega, indirect binary-n cube, baseline, and generalized cube.

A 2x2 switching element (SE) is the basic component of the SEN. The SE can either transmit (T) the inputs directly through itself or exchange (X) the inputs. A SEN is a unique path MIN [13, 12]. Therefore, there is only a single path between a particular input output pair. In this type of network, all SEs are critical and assumed as series connection. The SE can either transmit the inputs straight through it or has cross connections. The number of switches per stage, the number of links and the connection between stages are consistent. An eight input / eight-output SEN with three stages, 12 switches (SEs), and 32 links as shown in Fig.1.

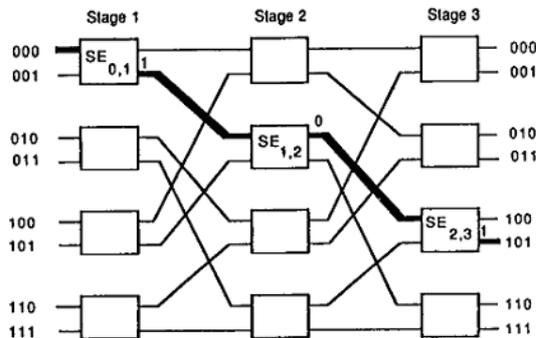


Figure 1: 8x8 Shuffle-Exchange Network (SEN).

A SEN+ is an NxN SEN with an additional stage. The SEN+ system has N inputs and N outputs, with two paths between each source-destination pair. It has  $n = \log_2 N + 1$  stages and each stage has N/2 SEs. In general, the switch complexity for the NxN SEN+ is  $N/2(\log_2 N + 1)$ . Thus, the additional cost of the SEN+ is N/2 switches or a fractional increase of  $1/\log_2 N$ , which is small for a large N. An example of the 8x8 SEN+ is demonstrated by Fig.2. The addition of an extra-stage to the SEN allows two paths for communication between each source and destination. While the paths in the first and the last stages of the SEN+ are not disjoint, the paths in the intermediate stages do disjoint links traverse. So

the path redundancy in the SEN+ is achieved at the expense of an additional stage to the SEN.

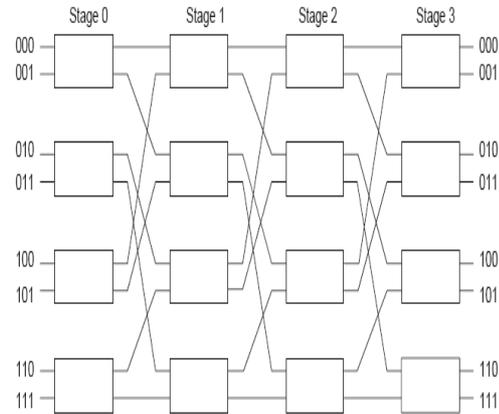


Figure 2: 8x8 SEN with an Extra-Stage (SEN+).

As a comparison to SEN and SEN+, a SEN with two additional stages (SEN+2) is presented [11], and the reliability is evaluated. In general, a SEN+2 consists of N inputs and N outputs, N/2 SEs per stage,  $\log_2 N + 2$  stages, and  $(N)(\log_2 N + 3)$  links. The network complexity is defined as the total number of SEs in the MIN, that is,  $(N/2)(\log_2 N + 2)$  which is 20 SEs for an 8x8 SEN+2 shown in Fig.3. The number of terminal paths between an input and an output switches will be increased to 2k by adding k extra stages to the SEN. This is also true for broadcast network. The additional k stages will create 2k broadcast paths between a particular source and all destinations. Therefore, a SEN is a  $(2k-1)$  fault tolerant. For the 8x8 case, the terminal paths and the broadcast paths of the SEN+ and SEN+2 are 2 and 4 respectively.

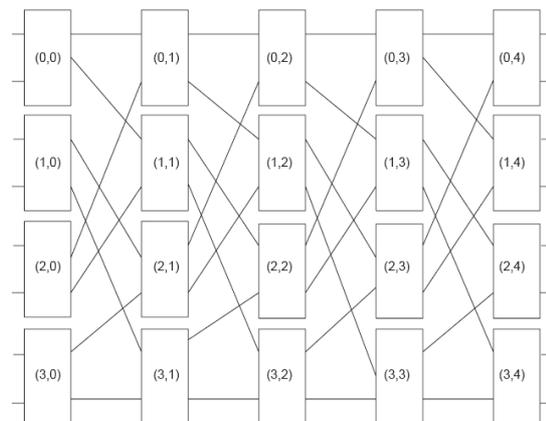


Figure 3: 8x8 Shuffle-Exchange Network with two Additional Stages (SEN+2)

### III. TERMINAL RELIABILITY OF SEN, SEN+, SEN+2

Terminal reliability is defined as the probability of successful communication between an input and an output switches. In this section, terminal reliability of SEN, SEN+ and SEN+2 is evaluated for the 8x8 network. The SEN is a unique-path MIN that has N input switches and N output switches and n stages, where  $n = \log_2 N$ . Each stage consists of N/2 interchange boxes, where each box being controlled individually through routing tags. An 8x8 SEN with three stages has 12 SEs and 32 links.

Let r be the probability of a switch being operational. Since these networks are unique-path MIN, the failure of any switch will cause system failure, so from the reliability point of view, there is  $\log_2 N$  SEs in series for each terminal path.

Hence, the terminal reliability of an NxN regular network is given by

$$R_t(\text{Network}) = (r)^{\log_2 N}$$

As there is only a single path between a particular input Si, i = 1, 2, 3, 4, and a particular output in the SEN so the terminal reliability for N = 8 is given by

$$R_t(\text{SEN}) = r^3$$

SEN+ is a two-path MIN derived from the SEN by adding an extra-stage. Figure 2 shows an 8x8 SEN+ with four stages consisting of 16 SEs and 40 links. Since the SEN+ is a two-path MIN, there are two distinct paths between a particular input and output pair. From the reliability point of view, this system can be represented as a parallel system path, consisting of  $(\log_2 N) - 1$  SEs each where each path is connecting the input and output SE in series. Hence the terminal reliability of an NxN SEN+ is given by

$$R_t(\text{SEN+}) = (r)^2(1 - (1 - r)^{(\log_2 N) - 1})^2$$

By adding an extra-stage to the 8x8 SEN, the number of connecting paths between any input and output switches will increase to two. Therefore, the terminal reliability of the 8x8 SEN+ is higher than that of the 8x8 SEN. From above equation, the terminal reliability of the SEN+ for N = 8 is given by

$$R_t(\text{SEN+}) = (r)^2(1 - (1 - r^2)^2) = 2(r)^2 - (r)^6$$

An 8x8 SEN+2 having four SEs per stage, five stages, and 48 links as demonstrated in Figure 3. It is observed that there are four terminal paths between any pair of input Si (i = 1, 2, 3, and 4) and output Di.

Suppose that the position of a SE i in stage j is represented by SE<sub>i,j</sub>. Since there are 20 SEs in the 8x8 SEN+2 and five stages (0, 1, 2, 3, and 4), the SEs are numbered from SE<sub>0,0</sub>, SE<sub>1,0</sub>, ..., SE<sub>2,4</sub>, SE<sub>3,4</sub>. As an example, the terminal reliability between SE<sub>0,0</sub> and SE<sub>0,4</sub> is examined as shown in Figure 4.3. The terminal reliability of the 8x8 SEN with two additional stages for N = 8 is

$$R_t(\text{SEN+2}) = r^2 \sum c_i r^{(2N-i)} (1-r)^i \quad \text{where } i = 0 \text{ to } 8$$

$$= r^{10} + 2r^9(1-r) + 8r^8(1-r)^2 + 8r^7(1-r)^3 + 2r^7(1-r)^2 + 4r^6(1-r)^3 + 4r^6(1-r)^2 + 4r^5(1-r)^2$$

### IV. EXPERIMENTAL RESULTS

The comparison of the SEN, SEN+ and SEN+2 for the 8x8 networks is presented in Table 4. From Figure 4 it is clear that the terminal reliability of the SEN+ is the highest whereas the terminal reliability of SEN+2 is the lowest among these three networks. Therefore, there is not a direct relation between additional paths and increase in terminal reliability because the additional paths may increase the links complexity of the network, leading to a higher failure. Hence, it is concluded that adding one additional stage to the SEN is more efficient way to improve terminal reliability rather than two stages.

Switching Reliability	Terminal Reliability of SEN	Terminal Reliability of SEN+	Terminal Reliability of SEN+2
0.99	0.970299	0.979712	0.924345
0.98	0.941192	0.958894	0.856787
0.96	0.884736	0.915935	0.742687
0.95	0.857375	0.893921	0.694677
0.94	0.830584	0.871628	0.651818
0.92	0.778688	0.826431	0.579273
0.90	0.729000	0.780759	0.520995

Table 4: Comparative Terminal reliability of SEN, SEN+ and SEN+2

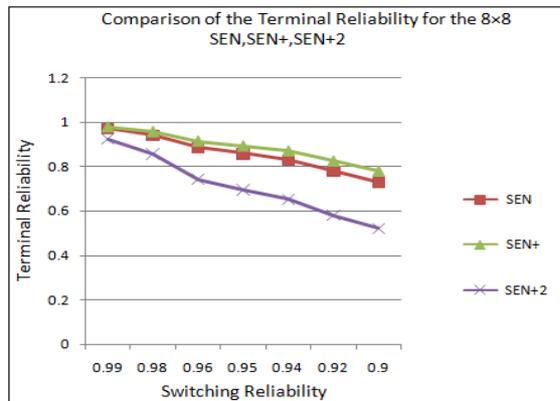


Figure 4: Terminal reliability graph of the 8x8 SEN, SEN+, SEN+2

## V. CONCLUSION

In this thesis Reliability analysis of regular multistage interconnection network namely SEN has been done. With the addition of one and two extra stages more regular MINs namely SEN+, SEN+2 are derived from SEN. As measures of network performance, the terminal reliability of all three networks have been evaluated. From the reliability analysis the following conclusion has been made:

Addition of one stage to any of SEN network provides higher reliability in terms of terminal reliability than the addition of two stages in the corresponding network.

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