

# Reliability Analysis of OMEGA Network and Its Variants

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## Abstract

The performance of a computer system depends directly on the time required to perform a basic operation and the number of these basic operations that can be performed concurrently. High performance computing systems can be designed using parallel processing. Parallel processing is achieved by using more than one processors or computers together they communicate with each other to solve a given problem. MINs provide better way for the communication between different processors or memory modules with less complexity, fast communication, good fault tolerance, high reliability and low cost. Reliability of a system is the probability that it will successfully perform its intended operations for a given time under stated operating conditions. From the reliability analysis it has been observed that addition of one stage to Omega networks provide higher reliability in terms of terminal reliability than the addition of two stages in the corresponding network.

**Keywords:** *Interconnection; Multistage; Networks; Reliability; Fault tolerant; Omega; Stages; Switching elements.*

## I. INTRODUCTION

Multistage interconnection networks (MINs) consist of more than one stages of small interconnection elements called switching elements and links interconnecting them. Multistage interconnection networks (MINs) are used in multiprocessing systems to provide cost-effective, high-bandwidth communication between processors and/or memory modules. A MIN normally connects  $N$  inputs to  $N$  outputs and is referred as an  $N \times N$  MIN. The parameter  $N$  is called the size of the network.

There are several different multistage interconnection networks proposed and studied in the literature. Figure1 illustrates a structure of multistage

interconnection network, which are representatives of a general class of networks. This Figure shows the connection between  $p$  inputs and  $b$  outputs, and connection between these is via  $n$  number of stages.

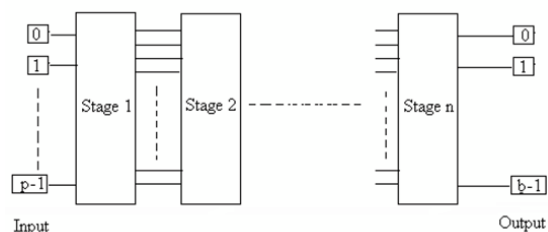


Figure 1: A Multistage Interconnection Network (MIN)

A multistage interconnection network is actually a compromise between crossbar and shared bus networks

Multistage interconnection networks are:

- Attempt to reduce cost
- Attempt to decrease the path length

In a multistage interconnection network, as in a crossbar, switching elements are distinct from processors. Instead messages pass through a series of switch stages.

The network can be constructed from unidirectional or bi-directional switches and links. In a unidirectional MIN, all messages must traverse the same number of wires, and so the cost of sending a message is independent of processor location. In effect, all processors are equidistant. In a bi-directional MIN, the number of wires traversed depends to some extent on processor location, although to a lesser extent than a mesh or hypercube [7].

**II. DESCRIPTION OF OMEGA, OMEGA+, OMEGA+2**

An N X N (N = 8) Omega network consists of  $n = \log_2 N$  stages of 2x2 switching elements (SEs) which provide connections between N sources and N destinations. A unique path between any source to a desired destination can be established by properly setting each SE to a state “through” or “cross.” An N- permutation defines N paths by specifying a distinct destination for each of the N sources.

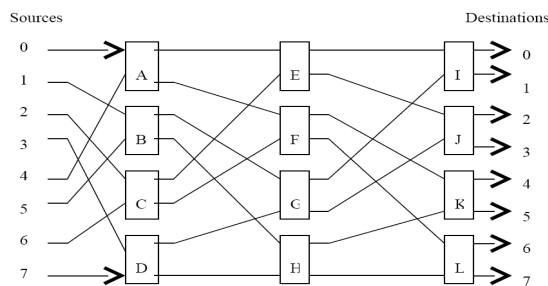


Figure 2: An Omega network for N=8

Two paths may conflict if they meet at a common output port of the same SE.

A k-Omega provides 2k different paths between any pair of source and destination and therefore provides fault tolerance ability and increases permutation capability. Figure 3 shows an example of a Omega with one additional stage.

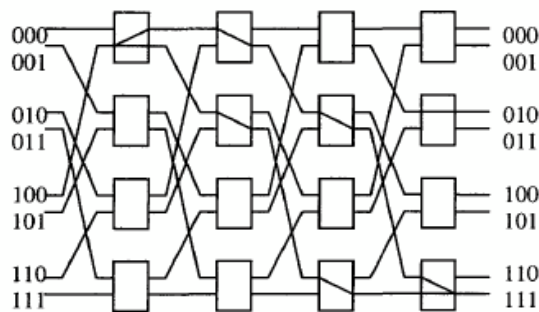


Figure 3: An Extra-Stage Omega (Omega+) Network for N=8

An Omega network with two additional stages (Omega+2) is presented in Figure 4 and the reliability of an 8x8 network is evaluated. In general, a Omega+2 consists of N inputs and N outputs, N/2 SEs per stage,  $\log_2 N + 2$  stages, and  $(N) (\log_2 N + 3)$

links. The network complexity is defined as the total number of SEs in the MIN, that is  $(N/2) (\log_2 N + 2)$  which is 20 switches for an 8x8 Omega+2.

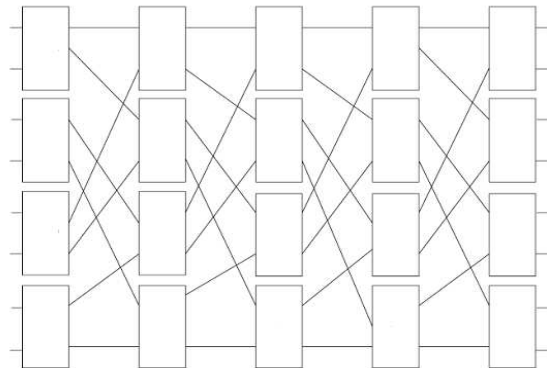


Figure 4: 8x8 Omega network with two additional stages (Omega+2).

**III. TERMINAL RELIABILITY OF OMEGA, OMEGA+, OMEGA+2**

Terminal reliability is defined as the probability of successful communication between an input output pair. In this section, terminal reliability of Omega, Omega+ and Omega+2 has evaluated. The Omega is a unique-path MIN that has N input switches and N output switches and n stages, where  $n = \log_2 N$ . An 8x8 Omega has three stages, 12 SEs and 32 links. Let r be the probability of a switch being operational. As Omega is a unique- path MIN, the failure of any switch will cause system failure, so from the reliability point of view, there are  $\log_2 N$  SEs in series for each terminal path. Hence, the terminal reliability of an NxN Omega is

$$R_t (\text{Omega}) = (r)^{\log_2 N}$$

As there is only a single path between a particular input  $S_i$ ,  $i = 1, 2, 3, 4$ , and a output in an 8x8 Omega so the terminal reliability is

$$R_t (\text{Omega}) = (r)^3.$$

Omega+ is a two-path MIN derived from the Omega by adding an extra-stage. Figure 4.5 shows an 8x8 Omega+ with four stages consisting of 16 SEs and 40 links. Since the Omega+ is a two-path MIN, there are two connection paths between a particular input and output. From the reliability point of view, this system can be represented as a parallel system path, consisting of  $\log_2 N - 1$  SEs each. Where, each path is connecting the input and output SE in series. Hence, the terminal reliability of an NxN Omega+ is

Switching Reliability	Terminal Reliability of Omega	Terminal Reliability of Omega+	Terminal Reliability of Omega+2
0.99	0.970299	0.981912	0.920174
0.98	0.941192	0.976894	0.879613
0.96	0.884736	0.917955	0.804785
0.95	0.857375	0.897928	0.764970
0.94	0.830584	0.889160	0.651869
0.92	0.778688	0.845987	0.5969825
0.90	0.729000	0.778547	0.5368793

$$R_t (\text{Omega+}) = (r)^2 (1 - (1 - r^{(\log_2 N) - 1})^2)$$

By adding an extra-stage to an Omega, the number of connecting paths between any input and output switches will increase to two. Therefore, the terminal reliability of an Omega+ is higher than that of an Omega. From above equation, the terminal reliability of the Omega+ for N=8 is

$$R_t (\text{Omega+}) = (r)^2 (1 - (1 - r^2)^2) = 2(r)^4 - (r)^6$$

An 8x8 Omega+2 consists of eight inputs and eight outputs, four SEs per stage, five stages, and 48 links. It is observed that there are four terminal paths between any pair of input and output.

Suppose that the position of a SE i in stage j is represented by SE<sub>i,j</sub>. Since there are 20 SEs in the 8x8 Omega+2 and five stages (0, 1, 2, 3, and 4), the SEs are numbered from SE<sub>0,0</sub>, SE<sub>1,0</sub>, ....., SE<sub>2,4</sub>, SE<sub>3,4</sub>. The terminal reliability of an Omega network with two additional stages for N = 8 is

$$R_t (\text{Omega+2}) = r^{10} + 2r^9 (1 - r) + 8r^8 (1 - r)^2 + 8r^7 (1 - r)^3 + 2r^7 (1 - r)^2 + 4r^6 (1 - r)^3 + 4r^6 (1 - r)^2 + 4r^5 (1 - r)^2$$

**IV. EXPERIMENTAL RESULTS**

The data values for terminal reliability of the Omega, Omega+ and Omega+2 networks are presented in Table 5. From Figure 6 it is clear that the terminal reliability of the Omega+ is the highest whereas terminal reliability of Omega+2 is the lowest among these three networks. Therefore, that there is not a direct relation between additional paths and increase in the terminal reliability because the additional paths may increase the links complexity of the network,

leading to a higher failure. Hence, it can be concluded that adding one additional stage to the Omega is more efficient way to improve terminal reliability rather than two stages.

Table 5: Comparative Terminal reliability of SEN, SEN+ and SEN+2

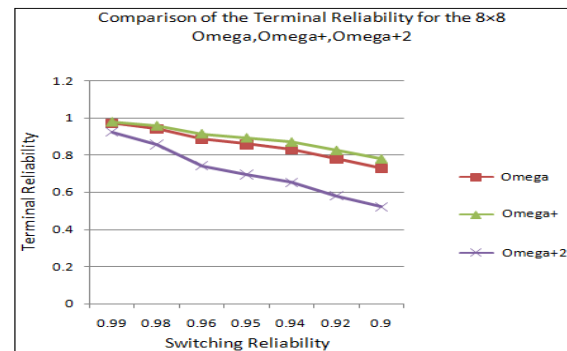


Figure 6: Terminal reliability graph of the 8x8 Omega, Omega+, Omega+2.

**V. CONCLUSION**

In this thesis Reliability analysis of regular multistage interconnection network namely OMEGA has been done. With the addition of one and two extra stages more regular MINs namely OMEGA +, OMEGA +2 are derived from OMEGA. As measures of network performance, the terminal reliability of all three networks have been evaluated. From the reliability analysis the following conclusion has been made: Addition of one stage to any of OMEGA network provides higher reliability in terms of terminal reliability than the addition of two stages in the corresponding network.

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