

# MARKOV CHAIN MODELING OF PERFORMANCE DEGRADATION OF PHOTOVOLTAIC SYSTEM

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## Abstract

Modern probability theory studies chance processes for which the knowledge of previous outcomes influence predictions for future experiments. In principle, when a sequence of chance experiments, all of the past outcomes could influence the predictions for the next experiment. In Markov chain type of chance, the outcome of a given experiment can affect the outcome of the next experiment. The system state changes with time and the state  $X$  and time  $t$  are two random variables. Each of these variables can be either continuous or discrete. Various degradation on photovoltaic (PV) systems can be viewed as different Markov states and further degradation can be treated as the outcome of the present state. The PV system is treated as a discrete state continuous time system with four possible outcomes, namely,  $s_1$  : Good condition,  $s_2$  : System with partial degradation failures and fully operational,  $s_3$  : System with major faults and partially working and hence partial output power,  $s_4$  : System completely fails. The calculation of the reliability of the photovoltaic system is complicated since the system have elements or subsystems exhibiting dependent failures and involving repair and standby operations. Markov model is a better technique that has much appeal and works well when failure hazards and repair hazards are constant. The usual practice of reliability analysis techniques include FMEA (failure mode and effect analysis), Parts count analysis, RBD (reliability block diagram), FTA (fault tree analysis) etc. These are logical, boolean and block diagram approaches and never accounts the environmental degradation on the performance of the system. This is too relevant in the case of PV systems which are operated under harsh environmental conditions. This paper is an insight into the degradation of performance of PV systems and presenting a Markov model of the system by means of the different states and transitions between these states.

**Keywords:** Markov chain, Derating, stochastic matrix, transition probability.

## I. INTRODUCTION AND LITERATURE REVIEW

The performance of photovoltaic systems varies with many environmental factors, viz, module temperature, ambient temperature, long term degradation, spectral issues, irradiance, wind speed, wind direction, air gap between modules, dust, rainfall, corrosion, water vapour intrusion, delamination of encapsulant materials, thermal

expansion, ultraviolet radiation, humidity, mechanical load, salt mist, partial shading, heat island impact, global climate change, summer-winter climate change, Staebler-Wronski effect, Clearness of sky, ageing and component derating. These factors cause degradation of the PV system during long term exposure to field. In a discrete approach four possible states can be considered for a PV system operating in real field, namely, good condition, system with partial degradation failures and fully operational, system with major faults and partially working and hence partial output power, and the system completely fails.

The literature review reveals that the most commonly used reliability analysis techniques can be grouped as follows.

*Quantitative:* The interval between the resulting numbers and the ratio of the resulting numbers has a meaning.

*Qualitative:* The resulting numbers are only used for distinction or rank ordering.

*Analysis by experts:* Based on previous experience in similar applications.

*FMEA (failure mode and effect analysis and derivatives):* Bottom-up analysis of a system, by examining all component failures and determining the effects of these failures on the entire system. *Parts count analysis or component count analysis:* An analysis technique to calculate the failure rate of a system when the failure rates of its components are known.

*RBD (reliability block diagrams):* a model of the behaviour of a system by showing graphically the condition for a successful operation.

*FTA (fault tree analysis):* top-down method, how basic events may lead to a certain top-event.

These are logical, boolean and block diagram approaches and never accounts the environmental degradation on the performance of the system. This is too relevant in the case of PV systems which are operated under harsh environmental conditions. This paper is an insight into the degradation of performance of PV systems and presenting a Markov model of the system by means of the different states and transitions between these states. Markov model (failure state diagram) is good tool in reliability analysis of electronic systems because the method is flexible and gives a realistic model. The method can include the following: • common cause failures • multiple failures • different repair times and • variable failure rates. Markov model is a state diagram model with circles and arrows. The circles represent the component states (working or failed), the arrows stand for the direction of transitions between the states (failure or repair), so the arrows are directed arcs. The failure or repair rates are presented by the arrows with numeric values. The component is in state 1, if it is successful, or in state 2, if it failed. The mode can move from state 1 to state 2 at a rate of  $\lambda_{12}$  (the failure rate), or from state 2 to state 1 at  $\mu_{21}$ (the repair rate).

In probability theory, a Markov model is a stochastic model that assumes the Markov property. Generally, this assumption enables reasoning and computation with the model that would otherwise be intractable. The field degradation is considered as non repairable and for model simplicity, the PV system is treated as a discrete state continuous time system with four possible outcomes, namely,  $s_1$  : Good condition,  $s_2$  : System with partial degradation failures and fully operational,  $s_3$  : System with major faults and partially working and hence partial output power,  $s_4$  : System completely fails.

A Markov chain can be described with the above states as follows. : Let there will be a set of *states*,  $S = \{s_1, s_2, s_3, \dots, s_r\}$ . The process starts in one of these states and moves successively from one state to another. Each move is called a *step*. If the chain is currently in state  $s_j$ , then it moves to state  $s_j$  at the next step with a probability denoted by  $p_{ij}$ , and this probability does not depend upon which states the chain was in before the current state. The probabilities  $p_{ij}$  are called *transition probabilities*. The process can remain in the state it is in, and this occurs with probability  $p_{ii}$ . An initial probability distribution, defined on  $S$ , specifies the starting state. Usually this is done by specifying a particular state as the starting state.

Up to now, failures resulting from degradation are not typically taken into consideration because of the difficulties in measuring the power of an individual module in a system. Photovoltaic (PV) modules are often considered as the most reliable element in PV systems. However, PV module reliability data are not shown on commercial datasheets in the same way as it is with other products such as electronic devices and electric power supplies. It is widely known that PV module

performance when deployed outdoors decreases steadily over time. After several years of operation this decrease will affect PV module reliability. Reliability evaluation based on degradation models is commonly applied in highly reliable products as a cost effective and confident way of evaluating their reliability. In this paper a degradation model for PV modules is presented and subsequently applied in the quantitative analysis of PV module reliability. With this model the different parameters related to module reliability such as the reliability function, failure rate function, the Mean time to failure (MTTF) or the warranty period can be assessed based on PV module degradation in the field.

## II. COMPONENT DERATING

Component derating is one of the major factor which reduces the reliability and efficiency of any PV system. The name normally given to operating a component well inside its normal operating limits, in order to reduce the rate at which the component deteriorates. Conceptually, it is easy to see that, the component may be specified to operate at high voltage and high temperature, applying those conditions simultaneously would probably be worse than applying either one or the other. Also reactions are known to proceed at higher speeds at higher temperatures, an insight originally shared by Arrhenius, one would predict

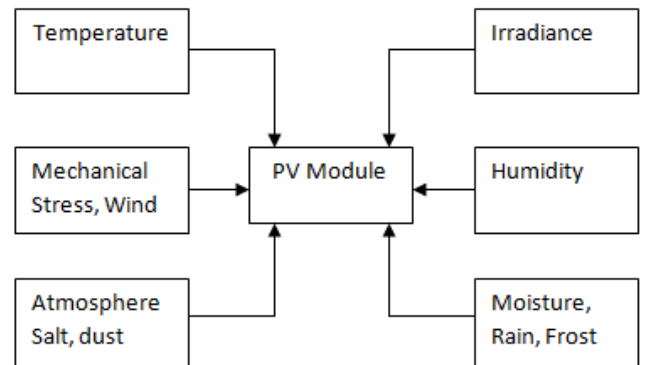


Figure 1: Various environmental stresses on PV module

reduced degradation, and hence extended life and enhanced reliability, by running a component at lower than its maximum category temperature.

Component	Standard	Low	High
Nameplate DC	0.950	0.880	1.050
Inverter and Transformer	0.920	0.880	0.980
Mismatch	0.980	0.970	0.995
Diodes and Connections	0.995	0.990	0.997
DC Wiring	0.980	0.970	0.990
AC Wiring	0.990	0.980	0.993

Soiling	0.950	0.300	0.995
Availability	0.980	0.000	0.995
Shading	1.000	0.000	1.000
Sun Tracking	1.000	0.950	1.350
Age	1.000	0.700	1.000
Overall DC-AC	0.770	0.09999	0.96001

TABLE 1 : STC Component Derating Factors of a PV System ( NREL and PVWATTS ).

### III. GENERAL MARKOV MODEL

Consider a non repairable system with components  $x_1, x_2, x_3$  and  $x_4$ , such that the system state is a function of the states of the components. The system is denoted by  $X$  and the system changes with time  $t$ , which are two random variables. There are four possible combinations, namely,  $\{X, t\}$  {(continuous state, continuous time), (discrete state continuous time), (continuous state, discrete time), (discrete state, discrete time)}. If the state of the system is probability based, then the model is a Markov probability model. A Markov chain can be described with the above states as follows. : Let there will be a set of states,  $S = \{s_1, s_2, s_3, \dots, s_r\}$ . The process starts in one of these states and moves successively from one state to another. Each move is called a *step*. If the chain is currently in state  $s_i$ , then it moves to state  $s_j$  at the next step with a probability denoted by  $p_{ij}$ , and this probability does not depend upon which states the chain was in before the current state. The probabilities  $p_{ij}$  are called *transition probabilities*. The process can remain in the state it is in, and this occurs with probability  $p_{ii}$ . An initial probability distribution, defined on  $S$ , specifies the starting state. Usually this is done by specifying a particular state as the starting state.

Let  $p_i(k)$  be the probability that the system  $S$  will be in state  $s_i$  ( $i = 1, 2, \dots, n$ ) after the  $k^{th}$  step and before the  $(k+1)^{th}$  step. The probabilities  $p_i(k)$  are called the probabilities of the Markov chain. After the  $k^{th}$  step, the system could be in any one of the  $n$  states. Hence,

$$\sum_{i=1}^n p_i(k) = 1 \quad \text{- Equation (1)}$$

The probability distribution of the states at the beginning of the process, i.e.,

$p_1(0), p_2(0), p_3(0), \dots, p_4(0), \dots, p_n(0)$ , - Equation (2) is

known as the initial probability distribution of the Markov chain. If the initial state  $S(0)$  of the system is known with certainty, say,  $S(0) = s_j$ , then the initial probability  $p_j(0) = 1$  and all other initial probabilities are zero. A Markov chain is said to be homogeneous if the transition probabilities  $P_{ij}$  depend only on from what step the system passes to which step, i.e.,

$$P_{ij} = P[S(k) = s_j | S(k-1) = s_i] \quad \text{- Equation (3)}$$

The transition probabilities  $P_{ij}$  of a homogeneous Markov chain form an  $n \times n$  matrix, called a transition matrix, given

by equation (4). The sum of the transition probabilities in any row of the matrix is equal to unity, i.e.,

$$\sum_{j=1}^n p_{ij} = 1 \quad (i = 1, 2, \dots, n) \quad \text{- Equation (4)}$$

A matrix which possesses the property given by equation (4) is known as a stochastic matrix. In equation (5),  $P_i$  is the probability that a system which is in state  $s_i$  before a given step will continue to remain in that state at the next step. The matrix equation (4) is a general transition matrix

$$[P_{ij}] = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1j} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2j} & \dots & P_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ P_{i1} & P_{i2} & \dots & P_{ij} & \dots & P_{in} \\ P_{n1} & P_{n2} & \dots & P_{nj} & \dots & P_{nn} \end{bmatrix}$$

- Equation (5)

Now consider a system for which the initial probability distributions, Equation (2) and the transition probabilities, Equation (5), are known. After the first step, the probability that the system is in state  $s_i$  ( $i = 1, 2, \dots, n$ ) is obtained from the total probability theorem equation. The total probability theorem states that,

$$P(S) = P[(K \text{ and } S) \text{ or } (\bar{K} \text{ and } S)] \text{-Equation (6)}$$

$$P(S) = P(K)P(S|K) + P(\bar{K})P(S|\bar{K})$$

$$P(S) = P(K)P(S|K) + P(\bar{K})P(S|\bar{K}) \quad \text{- Equation (7)}$$

The equation is applicable only for two events. It can be generalized as follows. If  $K_1, K_2, \dots, K_n$  be  $n$  mutually exclusive events which are collectively exhaustive and  $S$  is another event in the sample space, then the occurrence of  $S$  depends on the occurrence of  $K_1, K_2, \dots, K_n$ . Therefore,  $S = (S \text{ and } K_1) + (S \text{ and } K_2) + \dots + (S \text{ and } K_n)$ . Now,  $(S \text{ and } K_1), (S \text{ and } K_2), \dots (S \text{ and } K_n)$

$K_n$ ) are mutually exclusive since  $K_1, K_2, \dots, K_n$  are mutually exclusive.

Hence,  $P(S) = P(S \text{ and } K_1) + P(S \text{ and } K_2) + \dots + P(S \text{ and } K_n)$ .

$$P(S) = P(K_1)P(S|K_1) + P(K_2)P(S|K_2) + \dots + P(K_n)P(S|K_n).$$

$$P(S) = \sum_{i=1}^n P(K_i)P(S|K_i) \quad \text{--- Equation (8)}$$

Before the first step, the system can be in states  $s_1, s_2, \dots, s_n$  with probabilities  $p_1(0), p_2(0), \dots, p_n(0)$ . At the end of the first step, the system can be in states  $s_1, s_2, \dots, s_n$  with different probabilities. The system states before and after the first step is denoted by,  $s_1(0), s_2(0), \dots, s_n(0)$  and  $s_1(1), s_2(1), \dots, s_n(1)$  respectively.

The probability that the system is in state  $s_i(1)$  is,  $P[s_i(1)] = P[s_1(0) \text{ and } s_i(1)] + P[s_2(0) \text{ and } s_i(1)] + \dots + P[s_j(0) \text{ and } s_i(1)] + \dots + P[s_i(1)] = P[s_1(0)] P[s_i(1) | s_1(0)] + P[s_2(0)] P[s_i(1) | s_2(0)] + \dots + P[s_j(0)] P[s_i(1) | s_j(0)] + \dots + P[s_i(1)] = p_1(0)P_{1i} + p_2(0)P_{2i} + \dots + p_j(0)P_{ji} + \dots$

$$\text{Therefore } p_i(1) = \sum_{j=1}^n p_j(0)P_{ji} \quad \text{--- Equation (9)}$$

The probability that the system is in state  $s_j$  at the end of the  $k^{\text{th}}$  step, i.e.,  $s_j(k)$  depends on the probabilistic states of the system at the end of the  $(k-1)^{\text{th}}$  step. This is according to the assumption made in a Markov process.

$$P[s_j(k)] = P[s_1(k-1) \text{ and } s_j(k)] + P[s_2(k-1) \text{ and } s_j(k)] + \dots + P[s_j(k-1) \text{ and } s_j(k)] + \dots + P[s_j(k)] = P[s_1(k-1)] P[s_j(k) | s_1(k-1)] + P[s_2(k-1)] P[s_j(k) | s_2(k-1)] + \dots + P[s_j(k-1)] P[s_j(k) | s_j(k-1)] + \dots + P[s_j(k)] = p_1(k-1)P_{1j} + p_2(k-1)P_{2j} + \dots + p_j(k-1)P_{jj} + \dots \text{ Therefore,}$$

$$p_i(k) = \sum_{j=1}^n p_j(k-1)P_{ji} \quad \text{--- Equation (10)}$$

This general expression can be used to determine the probabilities of the states of the PV system.

#### IV. MARKOV MODELING OF DEGRADATION

In mathematics, a stochastic matrix (also termed probability matrix, transition matrix, or Markov matrix) is a matrix used to describe the transitions of a Markov chain. A right transition probability matrix is a square matrix each of whose rows consists of nonnegative real numbers, with each row summing to 1. The PV system is treated as a discrete state continuous time system with four possible outcomes, namely,  $s_1$ : Good condition,  $s_2$ : System with partial degradation failures and fully operational,  $s_3$ : System with major faults and partially working and hence partial output power,  $s_4$ : System completely fails. The transition probability

matrix is written based on the fact that a modern PV system should be capable to give satisfactory performance for around 30 years. This time is partitioned into four years each as, first 7 years : good and fully working, second 7 years : partial degradation and fully operational, third 7 years : major faults and partially working and last 7 years : complete failure.

Period	Y1	Y2	Y3	Y4
Beginning	0.9	0.06	0.04	0
Minor faults	0	0.5	0.3	0.2
Major faults	0	0	0.3	0.7
Complete failure	0	0	0	1

Table 2: Transition probability matrix of various faults ( Y1: Up to 7 years, Y2: 7 to 14 years, Y3 : 14 to 21 years, Y4 : 21 to 28 years.)

Assume the following probability, based on various field study that, there will be 1% degradation /year in field owing to various environmental factors, namely, module temperature, ambient temperature, long term degradation, spectral issues, irradiance, wind speed, wind direction, ageing and component derating, air gap between modules, dust, global climate change, summer-winter climate change, rainfall, corrosion, water vapour intrusion, delamination of encapsulant materials, Thermal expansion, ultraviolet radiation, humidity, mechanical load, salt mist etc.

The Markov directed graph of the four possible states can be constructed. Determine the probabilities of the defined states of the PV system after it undergoes one, two and three inspections. At the beginning the system is in good condition after installation. The transition matrix for the above probability is given by,

$$\begin{bmatrix} 0.9 & 0.06 & 0.04 & 0 \\ 0 & 0.5 & 0.3 & 0.2 \\ 0 & 0 & 0.3 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The four states of the PV system are,

$s_1$  : Good condition,  $s_2$  : System with partial degradation failures and fully operational,  $s_3$  : System with major faults and partially working and hence partial output power,  $s_4$  : System completely fails.

The directed graph is shown in figure x. To start with, the PV system is in good working condition. Thus,  $p_1(0) = 1$ .



From the directed graph, the probability that the system state is  $s_i$  ( $i = 1,2,3,4$ ) after the first inspection or after 7 years is,

$$P[s_1(1)] = p_1(1) = p_1(0)P_{11} = 1 \times 0.9 = 0.9,$$

$$P[s_2(1)] = p_2(1) = p_1(0)P_{12} = 1 \times 0.06 = 0.06,$$

$$P[s_3(1)] = p_3(1) = p_1(0)P_{13} = 1 \times 0.04 = 0.04,$$

$$P[s_4(1)] = p_4(1) = p_1(0)P_{14} = 1 \times 0 = 0.$$

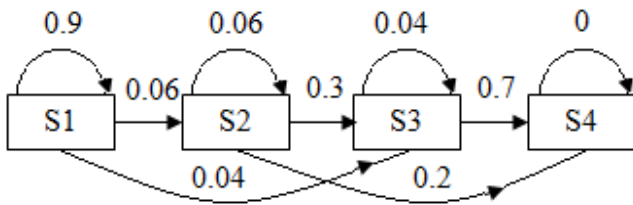


Figure 2: The directed graph of the PV system with four states from –No defect|| to –Complete failure||.

The states of the system after the first step can be obtained easily from the graph. But it is a good practice to use the general expression given in equation (10) for more convenient use. From equation (10),

$$p_i(k) = \sum_{j=1}^n p_j(k-1)P_{ji}.$$

For the second step,

$$p_i(2) = p_1(1)P_{1i} + p_2(1)P_{2i} + p_3(1)P_{3i} + p_4(1)P_{4i}.$$

$$p_1(2) = p_1(1)P_{11} + p_2(1)P_{21} + p_3(1)P_{31} + p_4(1)P_{41} = (0.9 \times 0.9) + (0.06 \times 0) + (0.04 \times 0) + (0 \times 0) = 0.81$$

$$p_2(2) = p_1(1)P_{12} + p_2(1)P_{22} + p_3(1)P_{32} + p_4(1)P_{42} = (0.9 \times 0.06) + (0.06 \times 0.5) + (0.04 \times 0) + (0 \times 0) = 0.084$$

$$p_3(2) = p_1(1)P_{13} + p_2(1)P_{23} + p_3(1)P_{33} + p_4(1)P_{43} = (0.9 \times 0.04) + (0.06 \times 0.3) + (0.04 \times 0.3) + (0 \times 1) = 0.066$$

$$p_4(2) = p_1(1)P_{14} + p_2(1)P_{24} + p_3(1)P_{34} + p_4(1)P_{44} = (0.9 \times 0) + (0.06 \times 0.2) + (0.04 \times 0.7) + (0 \times 1) = 0.04$$

For the third step, from equation (10),

$$p_i(3) = p_1(2)P_{1i} + p_2(2)P_{2i} + p_3(2)P_{3i} + p_4(2)P_{4i}.$$

Therefore,

$$p_1(3) = p_1(2)P_{11} + p_2(2)P_{21} + p_3(2)P_{31} + p_4(2)P_{41} = (0.81 \times 0.9) + (0.084 \times 0) + (0.066 \times 0) + (0.04 \times 0) = 0.729.$$

$$p_2(3) = p_1(2)P_{12} + p_2(2)P_{22} + p_3(2)P_{32} + p_4(2)P_{42} = (0.81 \times 0.06) + (0.084 \times 0.5) + (0.066 \times 0) + (0.04 \times 0) = 0.0906.$$

$$p_3(3) = p_1(2)P_{13} + p_2(2)P_{23} + p_3(2)P_{33} + p_4(2)P_{43} =$$

$$(0.81 \times 0.04) + (0.084 \times 0.3) + (0.066 \times 0.3) + (0.04 \times 0) = 0.0774.$$

$$p_4(3) = p_1(2)P_{14} + p_2(2)P_{24} + p_3(2)P_{34} + p_4(2)P_{44} = (0.81 \times 0) + (0.084 \times 0.2) + (0.066 \times 0.7) + (0.04 \times 1) = 0.103.$$

The above calculated results are the probabilities of the states of the PV system after it undergoes through the states,  $s_1$  : Good condition,  $s_2$  : System with partial degradation failures and fully operational,  $s_3$  : System with major faults and partially working and hence partial output power, and  $s_4$  : System completely fails.

## V. CONCLUSION AND FUTURE SCOPE

In designing a system, environmental parameters, must be specifically addressed to ensure that the design is robust. Two approaches that can be used to eliminate or mitigate the effects of variations in parameter values are: (1) Control the device and material parameter variations through process design and control to hold them within specified limits for a specified time under specified conditions. This will be referred to as Parts Control.

(2) Design circuits and systems to be sufficiently tolerant of variations in device and material parameters so that anticipated variations over time and stress do not degrade system performance. This will be referred to as Design Control.

The usual practice of reliability analysis is techniques like FMEA, RBD, FTA, Parts count analysis, etc. These are logical, boolean and block diagram approaches and never accounts the environmental degradation on the performance of the system. This is too relevant in the case of PV systems which are operated under harsh environmental conditions. The analysis using Markov model of the system will give the probability of failure of the system from one defined derating and degradation state to another. This will make the reliability prediction by accounting the environmental impacts at various periods so that the life time analysis and warranty fixation will be more performance oriented. The calculation of the reliability of the photovoltaic system is complicated since the system have elements or subsystems exhibiting dependent failures and involving repair and standby operations. Markov model is a better technique that has much appeal and works well when failure hazards and repair hazards are constant. The modeling can be extended by accounting more degradation stages so that a close microscopic information can be obtained.

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