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Statistical Analysis of Two Types Failure of Redundant System

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Abstract

In this paper we have analyzed a two unit stand by system in which the operative unit fails due to machinery defects as well as random shocks which occurs at random epochs of time.

Keywords: *Redundant System, States of the System, State Probabilities.*

1. Introduction

A large number of authors including [1, 2] have analyzed so many reliability models with the assumption that the failure of an operative unit occurs only due to its machinery defects. But in real practical situations there exists some systems in which the failure of an operative unit can also occurs due to random shocks.

In this paper, we are analyzing a two unit stand by system in which the operative unit fails due to machinery defects as well as random shocks which occurs at random epocns of time.

2. Assumptions

(i) The system consists of two non-identical units in which one unit is operative and other is cold standby.

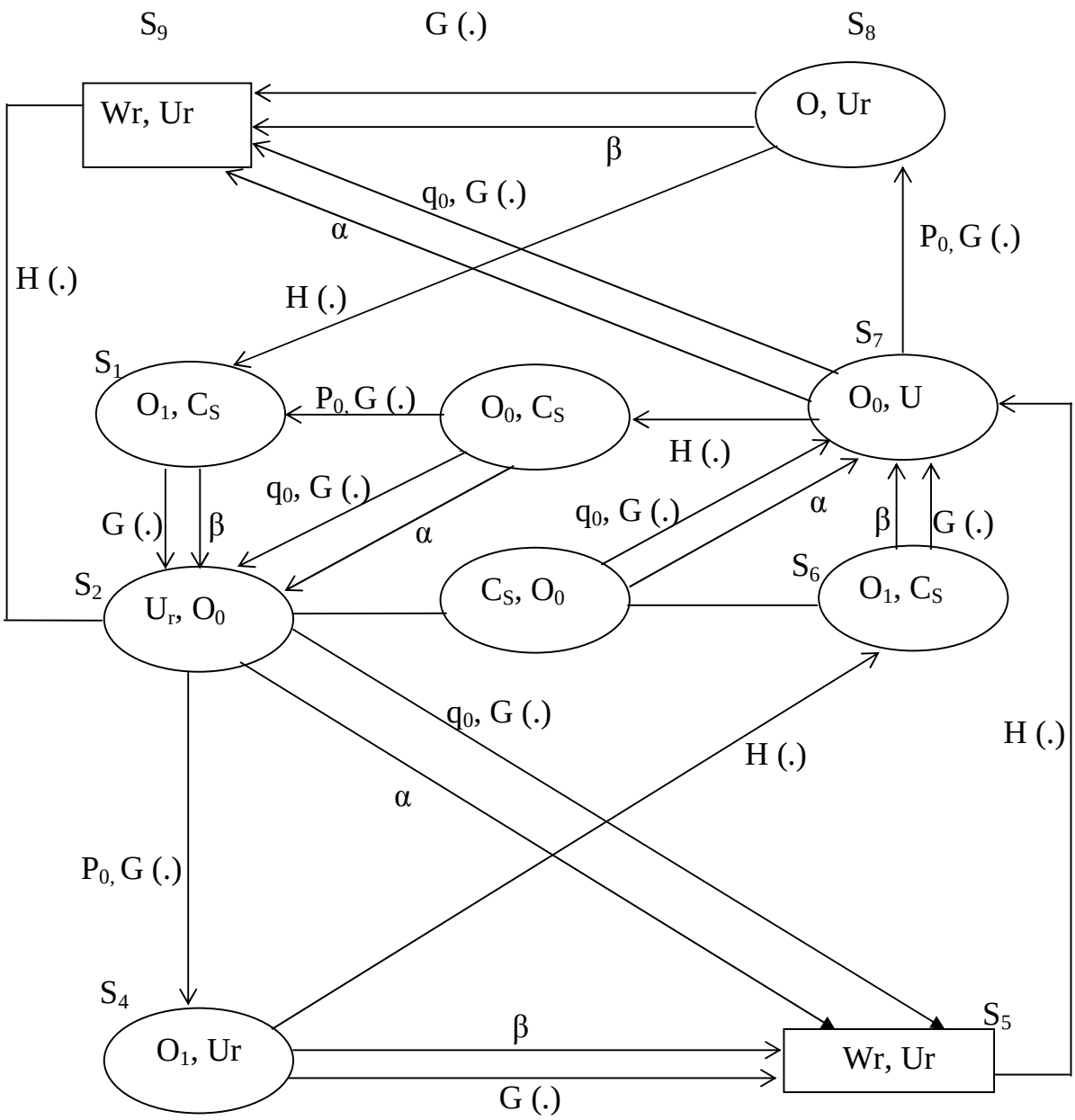
(ii) Upon failure of an operative unit the cold standby becomes operative instantaneously.

(iii) An operating unit can fail due to machine cause as well as due to the random shock s which occurs at random epoch of time.

(iv) An operating unit cannot sustain more than two shocks i.e. if the operating unit does not fail after first shock then it will definitely fail after second shock.

(v) The failure rate of an operative unit after experience the first shock increases automatically.

(vi) A single repair facility with discipline FCFS is used for repair. Identifying the suitable regenerative points the measures of system effectiveness discussed in paper-11 are obtained.



O: UP STATE : DOWN STATE

Fig. 1

3. Notation and States of the System

O_O	: -	Operating unit.
O_1	:-	Operating unit after experienced first shock
C_S	: -	Cold standby unit
U_R	: -	Failed unit under repair
W_R	: -	Failed unit waiting for repair
α	:-	Constant failure rate of operating unit O_o
β	:-	Constant failure rate of operating unit O_1
g(.), G(.):-	p.d.f and c.d.f, of time for occurring random shocks	
h(.), H(.):-	p.d.f and c.d.f, of repair time.	
$P_o(1-q_o)$:-	Probability that unit is not failed in first shock.	

Possible states of the system are
Up states:-
S_o:(O_o, C_s), S_1:(O_1,C_s), S_2:(U_R, O_o), S_3:(C_s,O_o)
S_4:(U_R, O_1), S_6:(C_s,O_1), S_7:(O_o, U_R), S_8:(O_1,U_R)

Down states:-
S_5:(U_R, U_R), S_9:(Co(W_R, U_R)
Possible states and transitions are shown fig. 1

4. Transition and Steady State Probabilities

In fig. 4.1 the epochs of entry into states S_0, S_1, S_2, S_3, S_4, S_6, S_7 and S_8 are regenerative points and E is the set of these states.

Using the definition of $Q_1(t)$, the t.p.m. of embedded Markov chain is

$$P \equiv \left(P_{ij}\right) \equiv \left[Q_i, (\infty)\right] \tag{1}$$

With non-zero elements.

$$Q_{01}(t) = \int_0^t P_o o^{-\alpha u} g(u) du;$$

$$Q_{02}(t) = \int_0^t \left[\alpha e^{-\alpha u} G(u) + q_o \Theta^{-\alpha u} g(u) du\right]$$

$$Q_{12}(t) = \int_0^t \left[\beta e^{-\beta u} G(u) + \Theta^{-\beta u} g(u)\right] du$$

$$Q_{23}(t) = \int_0^t e^{-\alpha u} G(u) h(u) du$$

$$Q_{24}(t) = \int_0^t e^{-\alpha u} G(u) \bar{h}(u) du$$

$$Q_{25}(t)=\int_0^t\left[\alpha e^{-\alpha u}\,\bar{G}(u)\,\tilde{H}(u)+q_o\Theta^{-\alpha u}g(u)\,\bar{H}(u)\right]du$$

$$Q_{36}(t)=\int_0^t P_o e^{-\alpha u}g(u)du$$

$$Q_{37}(t)=\int_0^t\left[\alpha e^{-\alpha u}\,\bar{G}(u)+q_o\Theta^{-\alpha u}g(u)\right]du$$

$$Q_{45}(t)=\int_0^t\left[\beta e^{-\beta u}\,\bar{G}(u)\,\bar{H}(u)+e^{-\beta u}g(u)\,\bar{H}(u)\right]du$$

$$Q_{46}(t)=\int_0^t e^{-\beta u}\,\bar{G}(u)\,\bar{h}(u)du$$

$$Q_{57}(t)=\int_0^t h(u)du$$

$$Q_{67}(t)=\int_0^t\left[\beta e^{-\beta u}\,\bar{G}(u)+e^{-\beta u}g(u)\right]du$$

$$Q_{70}(t)=\int_0^t e^{-\alpha u}\,\bar{G}(u)h(u)du$$

$$Q_{78}(t)=\int_0^t P_o\Theta^{-\alpha u}g(u)\,\bar{H}(u)du$$

$$Q_{79}(t)=\int_0^t\left[\alpha e^{-\alpha u}\,\bar{G}(u)\,\bar{H}(u)+q_o e^{-\alpha u}g(u)\,\bar{H}(u)\right]du$$

$$Q_{81}(t)=\int_0^t\left[e^{-\beta u}\,\bar{G}(u)h(u)\right]du$$

$$Q_{89}(t)=\int_0^t\left[\beta e^{-\beta u}\,\bar{G}(u)\,\bar{H}(u)+e^{-\beta u}g(u)\,\bar{H}(u)\right]du \qquad (2\text{-}19)$$

$$Q_{92}(t)=\int_0^t h(t)dt$$

The transition probability $Q_{2,7}$ (t) is the product of the following probabilities

(i) Probability that the system passes from state S_2 to S_5 during time (U, U + du), U<t which is equivalent tQ $\int_0^t\left[\alpha e^{-\alpha u}\,\bar{G}(u)\,\bar{H}(u)+q_o e^{-\alpha u}g(u)\,\bar{H}(u)\right]du$

(ii) Probability that system passes from state S_5 to state S_7 during the interval (x, x + dx) in (u, t) which is equivalent to $\int_0^t dH(x)/\bar{H}(u)$ thus the expression for $Q_2^{(5)},7(t)$ is

$$Q_{2,7}^{(5)}(t)=\int_0^t\left[\alpha e^{-\alpha u}\,\bar{G}(u)\,\bar{H}(u)+q_o\Theta^{-\alpha u}g(u)\,\bar{H}(u)\right]du\int_0^t dH(x)/\bar{H}(u)$$

$$=\int_0^t\left[\alpha e^{-\alpha u}\,\bar{G}(u)+q_o\Theta^{-\alpha u}g(u)\right]du\int_0^t dH(x)$$

Change the order of integration gives,

$$=\int_0^t dH(x)\int_0^t\left[\alpha e^{-\alpha u}\,\bar{G}(u)+q_o e^{-\alpha u}g(u)\right]du$$

$$Q_{2,7}^{(5)}(t)=\alpha\left[\int_0^t dH(x)\int_0^x e^{-\alpha u}\,\bar{G}(u)du\right]+q_o\left[\int_0^t dH(x)\int_0^x e^{-\alpha u}g(u)du\right]$$

$$=\int_0^t dH(x)-P_o\int_0^t e^{-\alpha x}g(x)dx-\int_0^t e^{-\alpha x}h(x)dx+\alpha\int_0^t e^{-\alpha x}G(x)H(x)dx-q_o\int_0^t e^{-\alpha x}g(x)H(x)dx)$$

20

Similarly, we can obtain expression for

$$Q_{4,7}^{(5)}(t)=\int_0^t dH(x)-\int_0^x e^{-\beta x}h(x)dx-\int_0^t e^{-\beta x}g(x)H(x)dx+\beta\int_0^t e^{-\beta x}G(x)H(x)dx$$

$$Q_{7,2}^{(9)}(t)=\int_0^t dH(x)-P_o\int_0^t \Theta^{-\alpha x}g(x)dx-\int_0^t \Theta^{-\alpha x}h(x)dx+\alpha\int_0^t e^{-\alpha x}G(x)H(x)dx-q_o\int_0^t e^{-\alpha x}g(x)H(x)dx$$

$$Q_{8,2}^{(9)}(t)=\int_0^t dH(x)-\int_0^t e^{-\beta x}h(x)dx+\beta G(x)H(x)dx-\int_0^t e^{-\beta x}g(x)H(x)dx$$

(21-23)

From the above transition probabilities (21-23), we can obtain the steady state probabilities p_{ij}'s by taking the limit as t-->∞. Thus,

$$P_{o1}=P_o g^*(\alpha)$$

$$P_{o2}=1-P_o g^*(\alpha)$$

$$P_{23}=h^*(\alpha)-\int e^{-\alpha t}G(t)H(t)dt$$

$$P_{24}=P_o[g^*(\alpha)-\int e^{-\alpha t}g(t)H(t)dt]$$

$$P_{28}=1-P_o g^*(\alpha)-h^*(\alpha)+\alpha\int e^{-\alpha t}G(t)H(t)dt-q_o\int e^{-\alpha t}g(t)H(t)dt=P_{27}^{(5)}$$

$$P_{36}=P_o g^*(\alpha)$$

$$P_{37}=1-P_o g^*(\alpha)$$

$$P_{45} = 1 - h^*(\beta) - \int e^{-\beta t} g(t)H(t)dt + \beta \int e^{-\beta t} G(t)H(t)dt = P_{47}^{(5)}$$

$$P_{46} = h^*(\beta) - \int e^{-\beta t} G(t)h(t)dt$$

$$P_{70} = h^*(\alpha) - \int e^{-\alpha t} G(t)h(t)dt$$

$$P_{75} = P_o g^*(\alpha) - P_o \int e^{-\alpha t} g(t)h(t)dt$$

$$P_{77} = i - P_o g^*(\alpha) - h^*(\alpha) + \alpha \int e^{-\alpha t} G(t)H(t)dt - q_o \int e^{-\alpha t} g(t)H(t)dt = P_{72}^{(9)}$$

$$P_{81} = h^*(\beta) - \int e^{-\beta t} G(t)h(t)dt$$

$$P_{89} = i - h^*(\beta) + \beta \int e^{-\beta t} G(t)h(t)dt - \int e^{-\beta t} g(t)h(t)dt = P_{92}^{(9)}$$

(24-37)

$P_{12}=P_{57}=P_{67}=P_{92} = 1$

$P_{01}+ P_{02} = 1$

$P_{23}+ P_{24}+ P_{25}= 1= P_{23}+ P_{24}+ P_{27}^{(5)}$

$P_{36}+ P_{37} = 1$

$P_{46}+ P_{45} = 1 = P_{46}+ P_{47}^{(5)}$

$P_{70}+ P_{78}+ P_{79}= 1= P_{70}+ P_{78}+ P_{72}^{(9)}$

$P_{81}+ P_{89} = 1= P_{81}+ P_{82}^{(9)}$

(38-44)

5. Mean Sojourn Times

(a) As defined earlier, the mean sojourn time in state S_i € E is $\mu_1 = E(T) = \int P[T > t]dt$

(45)

Using this we obtain the following relations:

$$\mu_o = [1 - g^*(\alpha)]/\alpha \quad ; \quad \mu_1 = [1 - g^*(\beta)]/\beta$$

$$\mu_2 = \int e^{-\alpha t} \bar{G}(t) \bar{H}(t) dt \quad ; \quad \mu_3 = [1 - g^*(\alpha)]/\beta$$

$$\mu_4 = \int e^{-\beta t} \bar{G}(t) \bar{H}(t) dt \quad ; \quad \mu_6 = [1 - g^*(\beta)]/\beta$$

$$\mu_7 = \int e^{-\alpha t} \bar{G}(t) \bar{H}(t) dt \quad ; \quad \mu_8 = \int e^{-\beta t} \bar{G}(t) \bar{H}(t) dt$$

(46-53)

b) The contribution to mean Sojourn time in state Si when system transits direct to S_j is

$$mij = \int t dQ_1,(t) = -\int Q_1,(t)dt$$

(54)

Thus

$$m_{o1} = P_o \int te^{-\alpha t} g(t)dt$$

$$m_{o2} = \int \left[t\alpha e^{-\alpha t} \bar{G}(t) + q_o + \Theta^{-\alpha t} g(t) \right] dt$$

$$m_{12} = \int \left[t\beta e^{-\beta t} \bar{G}(t) + te^{-\beta t} g(t) \right] dt$$

$$m_{23} = \int \left[te^{-\alpha t} \bar{G}(t) h(t) \right] dt$$

$$m_{24} = P_o \int te^{-\alpha t} g(t) + \bar{H}(t) dt$$

$$m_{25} = \int \left[t\alpha e^{-\alpha t} \bar{G}(t) \bar{H}(t) + tq_o e^{-\alpha t} g(t) \bar{H}(t) \right] dt$$

$$m_{36} = \int tP_o e^{-\alpha t} g(t) dt$$

$$m_{37} = \int \left[t\alpha e^{-\alpha t} \bar{G}(t) + tq_o e^{-\alpha t} g(t) \right] dt$$

$$m_{45} = \int \left[t\beta e^{-\beta t} \bar{G}(t) \bar{H}(t) + te^{-\beta t} g(t) \bar{H}(t) \right] dt$$

$$m_{46} = \int te^{-\beta t} \bar{G}(t) h(t) dt$$

$$m_{67} = \int \left[t\beta e^{-\beta t} \bar{G}(t) + te^{-\beta t} g(t) \right] dt$$

$$m_{70} = \int te^{-\alpha t} \bar{G}(t) h(t) dt$$

$$m_{78} = \int P_o te^{-\alpha t} g(t) \bar{H}(t) dt$$

$$m_{79} = \int \left[\alpha e^{-\alpha t} \bar{G}(t) \bar{H}(t) + tq_o e^{-\alpha t} g(t) \bar{H}(t) \right] dt$$

$$m_{81} = \int te^{-\alpha t} \bar{G}(t) h(t) dt$$

$$m_{89} = \int \left[t\beta e^{-\beta t} \bar{G}(t) \bar{H}(t) + te^{-\beta t} g(t) \bar{H}(t) \right] dt$$

$$m_{92} = \int th(t) dt$$

$$m_{27}^{(5)} = \int t\left(1 - e^{-\alpha t}\right) dH(t) - P_o \int te^{-\alpha t} g(t) dt + \alpha \int te^{-\alpha t} G(t) H(t) dt - q_o \int te^{-\alpha t} g(t) H(t) dt$$

$$m_{47}^{(5)} = \int t\left(1 - e^{-\beta t}\right) dH(t) - \int te^{-\beta t} g(t) H(t) dt + \beta \int te^{-\beta t} G(t) H(t) dt$$

$$m_{72}^{(a)} = \int t\left(1 - e^{-1\alpha t}\right) dH(t) - P_o \int te^{-\alpha t} g(t) dt + \alpha \int te^{-\alpha t} G(t) H(t) dt - q_o \int te^{-\alpha t} g(t) H(t) dt$$

$$m_{82} = \int t\left(1 - e^{-\beta t}\right) dH(t) + \beta \int te^{-\beta t} G(t) H(t) dt - \int te^{-\beta t} g(t) H(t) dt$$

(55-75)

Relation (55-75) satisfy the following

$$m_{01} + m_{02} = [1 - g^*(\alpha)]/\alpha = \mu_o$$
$$m_{12} = [1 - g^*(\beta)]/\beta = \mu_1$$
$$m_{23} + m_{24} + m_{25} = \int e^{-\alpha t}\, \bar{G}(t)\, \bar{H}(t) dt = \mu_2$$
$$m_{36} + m_{37} = [1 - g^*(\alpha)]/\alpha = \mu_3$$
$$m_{45} + m_{46} = \int e^{-\beta t}\, \bar{G}(t)\, \bar{H}(t) dt = \mu_4$$
$$m_{67} = [1 - g^*(\beta)]/\beta = \mu_6$$
$$m_{70} + m_{79} + m_{79} = \int e^{-\alpha t}\, \bar{G}(t)\, \bar{H}(t) dt$$
$$m_{81} + m_{89} = \int e^{-\beta t}\, \bar{G}(t)\, \bar{H}(t) dt$$
$$m_{23} + m_{24} + m_{27}^{(5)} = m_1$$
$$m_{47}^{(5)} + m_{46} = m_2$$
$$m_{70} + m_{78} + m_{72}^{(9)} = m_3$$
$$m_{82}^{(9)} + m_{81} = m_4$$

(76-87)

6. Mean Time to System Failure

TQ investigate the distribution of function $\pi_i(t)$ of T.S.F. with starting state S_i, the failed S_5 and S_9 are regarded as absorbing

Using the probabilities arguments the recursive relations among $\pi_i(t)$ are:-

$\Pi_o(t) = Q_{o1}(t)\ \$\ \pi_1(t) + Q_{o2}(t)\$\ \pi_2(t)$

$\pi_1(t) = Q_{12}(t)\ \$\ \pi_2(t)$

$$\pi_2(t) = Q_{23}(t)\$\pi_3(t) + Q_{24}(t)\$\pi_4(t) + Q_{25}(t)$$
$$\pi_3(t) = Q_{36}(t)\$\pi_6(t) + Q_{37}(t)\$\pi_7(t)$$
$$\pi_4(t) = Q_{45}(t) + Q_{46}(t)\$\pi_6(t)$$
$$\pi_6(t) = Q_{67}(t)\$\pi_7(t)$$
$$\pi_7(t) = Q_{70}(t)\$\pi_0(t) + Q_{78}(t)\$\pi_8(t) + Q_{79}(t)$$
$$\pi_8(t) = Q_{81}(t)\$\pi_1(t) + Q_{89}(t)$$

(88-95)

Taking Laplace-stieltyes transform of these equations, we get

$$\tilde{\pi}_o(s) = \tilde{Q}_{o1}(s)\tilde{\pi}_1(s) + \tilde{Q}_{02}(s)\tilde{\pi}_2(s)$$
$$\tilde{\pi}_1(s) = \tilde{Q}_{12}(s)\tilde{\pi}_2(s)$$
$$\tilde{\pi}_2(s) = \tilde{Q}_{23}(s)\tilde{\pi}_3(s) + \tilde{Q}_{24}(s)\tilde{\pi}_4(s) + \tilde{Q}_{25}(s)$$
$$\tilde{\pi}_3(s) = \tilde{Q}_{36}(s)\tilde{\pi}_4(s) + \tilde{Q}_{37}(s)\tilde{\pi}_7(s)$$
$$\tilde{\pi}_4(s) = \tilde{Q}_{45}(s) + \tilde{Q}_{46}(s)\tilde{\pi}_6(s)$$
$$\tilde{\pi}_6(s) = \tilde{Q}_{67}(s)\tilde{\pi}_7(s)$$
$$\tilde{\pi}_7(s) = \tilde{Q}_{7o}(s)\tilde{\pi}_0(s) + \tilde{Q}_{78}(s)\tilde{\pi}_8(s) + \tilde{Q}_{79}(s)$$
$$\tilde{\pi}_9(s) = \tilde{Q}_{81}(s)\tilde{\pi}_1(s) + \tilde{Q}_{89}(s)$$

(96-103)

Solving the above equations for $\tilde{\pi}_o(s)$ after omitting the argument "S" for brevity, we get

$$\tilde{\pi}_o(s) = N_1(s) \mid D_1(s) \tag{104}$$

Where

$$N_1(s) = \left(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\right)\left(\tilde{Q}_{23}\tilde{Q}_{36} + \tilde{Q}_{67} + \tilde{Q}_{24}\tilde{Q}_{46} + \tilde{Q}_{67} + \tilde{Q}_{23}\tilde{Q}_{37}\right)$$
$$\left(\tilde{Q}_{78}\tilde{Q}_{89} + \tilde{Q}_{79}\right) + \left(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\right)\left(\tilde{Q}_{24}\tilde{Q}_{45} + \tilde{Q}_{25}\right)$$

and

$$D_1(s) = \left(\tilde{Q}_{78}\tilde{Q}_{81} + \tilde{Q}_{12} + \tilde{Q}_{70}\tilde{Q}_{01} + \tilde{Q}_{12} + \tilde{Q}_{70}\tilde{Q}_{02}\right) \tag{105}$$

$$N_1(s) = \left(\tilde{Q}_{23}\tilde{Q}_{37} + \tilde{Q}_{23}\tilde{Q}_{67} + \tilde{Q}_{24}\tilde{Q}_{46}\tilde{Q}_{67}\right) \tag{106}$$

Taking limit as S→0 in (104), we get,

$$\tilde{\pi}_o(o) = 1$$

This shows that $\pi_0(t)$ is a proper c.d.f. Thus, MTSF of the system when it starts operation from S_0 is

$$E(T) = \frac{-d\,\bar{\pi}_0(s)}{ds} \mid s = o$$
$$= \frac{D_1'(o) - N_1'(o)}{D_1'(o)} = N_1 / D_1 \tag{107}$$

Where,

$$N_1 = \mu_o + P_{o1}\mu_1 + \mu_2 + P_{23}\mu_3 + P_{24}\mu_4 + \left(P_{23} + P_{24}P_{46}\right)$$
$$\left(P_{78}P_{02}P_{81}\mu_1 - P_{78}P_{81}\mu_o + \mu_7 + \mu_8 P_{78}\right) + \left(P_{23}P_{36} + P_{24}P_{46}\mu_6\right) \quad (108)$$

and

$$D_1 = \left(P_{25} + P_{24}P_{45}\right) + \left(P_{78}P_{89} + P_{79}\right)\left(P_{23}P_{37} + P_{24}P_{46} + P_{23}P_{36}\right) \quad (109)$$

7. Availability Analysis

As defined in section 2.7, the expression for $M_1(t)$ are:-

$$M_o(t) = e^{-\alpha t}\,\bar{G}(t) \quad ; \quad M_1(t) = e^{-\beta t}\,\bar{G}(t)$$
$$M_2(t) = e^{-\alpha t}\,\bar{H}(t)\bar{G}(t) \quad ; \quad M_3(t) = e^{-\alpha t}\,\bar{G}(t)$$
$$M_4(t) = e^{-\beta t}\,\bar{H}(t)\bar{G}(t) \quad ; \quad M_6(t) = e^{-\beta t}\,\bar{G}(t)$$
$$M_7(t) = e^{-\alpha t}\,\bar{H}(t)\bar{G}(t) \quad ; \quad M_8(t) = e^{-\beta t}\,\bar{H}(t)\bar{G}(t)$$

(110-117)

After taking Laplace transform

$$M_o^*(t) = \mu_o; \quad M_1^*(t) = \mu_1; \quad M_2^*(t) = \mu_2; \quad M_3^*(t) = \mu_3$$
$$M_4^*(t) = \mu_4; \quad M_6^*(t) = \mu_6; \quad M_7^*(t) = \mu_7; \quad M_8^*(t) = \mu_8 \quad (118\text{-}125)$$

From the argument used in the theory of regenerative process, the point-wise availability $A_1(t)$ are seen to satisfy the following recursive relations:

$$A_o(t) = M_o(t) + q_{01}(t)\ell A_1(t) + q_{02}(t)\ell A_2(t)$$
$$A_1(t) = M_1(t) + q_{12}(t)\ell A_2(t)$$
$$A_2(t) = M_2(t) + q_{23}(t)\ell A_3(t) + q_{24}(t)\ell A_4(t) + q_{27}^{(s)}(t)\ell A_7(t)$$
$$A_3(t) = M_3(t) + q_{36}(t)\ell A_6(t) + q_{37}(t)\ell A_7(t)$$
$$A_4(t) = M_4(t) + q_{46}(t)\ell A_6(t) + q_{47}^{(s)}(t)\ell A_7(t)$$
$$A_6(t) = M_6(t) + q_{67}(t)\ell A_7(t)$$
$$A_7(t) = M_7(t) + q_{10}(t)\ell A_o(t) + q_{78}(t)\ell A_8(t) + q_{72}^{(a)}(t)\ell A_2(t)$$
$$A_8(t) = M_8(t) + q_{81}(t)\ell A_1(t) + q_{82}^{(a)}(t)\ell A_2(t)$$

(126-133)

Taking Laplace transform of (126-133) a system of linear equations in $A_1^*(s)$ can be expressed in the matrix from (omitting the argument "s" for brevity) as $\left(A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_6^*, A_7^*, A_8^*\right)$

$$= Q - 1\left(M_0^*, M_1^*, M_2^*, M_3^*, M_4^*, M_6^*, M_7^*, M_8^*\right) \quad (134)$$

Where

$$Q = \begin{pmatrix} 1 & -q_{01}^{*} & -q_{02}^{*} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -q_{12}^{*} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{23}^{*} & -q_{24}^{*} & 0 & -q_{27}^{*(5)} & 0 \\ 0 & 0 & 0 & 1 & 0 & -q_{36}^{*} & -q_{37}^{*} & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{46}^{*} & -q_{47}^{*(5)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -q_{67}^{*} & 0 \\ -q_{70}^{*} & 0 & -q_{72}^{*(a)} & 0 & 0 & 0 & 1 & -q_{78}^{*} \\ 0 & -q_{81}^{*} & -q_{82}^{*} & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Computing the relevant cofactors of first column elements of Q, the solution of (134) gives the Laplace transform to point-wise availability $A_o(t)$ as

$$A_0^{*}(s) = N_2(s) / D_2(s) \tag{135}$$

Where

$$N_2(s) = M_o(t) + q_{01}(t)M_1(t) + \{q_{23}(t)[q_{36}(t) + Q_{67}(t) + q_{37}(t)] + q_{24}(t)(q_{46}(t)q_{67}(t) + q_{47}^{(5)}(t) + q_{27}^{(5)}(t)]$$
$$\left[\begin{matrix}(q_{01}(t)q_{12}(t) + q_{02}(t))(M_7(t) + q_{78}(t)M_8(t) + q_{78}(t)q_{81}(t)M_1(t)) - q_{78}(t)(q_{81}(t))(q_{81}(t)q_{12}(t) + q_{72}^{(a)}(t)) \\ (M_o(t) + q_{01}(t)M_1(t))\end{matrix}\right]$$
$$+ q_{01}(t)M_1(t)] + (q_{01}(t)q_{12}(t) + q_{02}(t))[M_2(t) + q_{23}(t)(M_3(t) + q_{36}(t)M_6(t)) + q_2(t)(M_4(t) + q_{46}(t)M_6(t))] \tag{136}$$

And

$$D_2(s) = 1 - [q_{23}(t)\{q_{36}(t)q_{67}(t) + q_{37}(t)\} + q_{24}(t)\{q_{46}(t)q_{67}(t)\} + q_{47}^{(5)} + q_{27}^{(5)}]$$
$$[q_{78}(t)(q_{81}(t)q_{12}(t) + q_{82}^{(9)}(t)) + q_{72}^{(9)}(t) + (q_{01}(t)q_{12}(t) + q_{02}(t))q_{70}(t)] \tag{137}$$

The steady state availability when the system starts operations from S_o is thus obtained as follows

$$A_o(\infty) = \lim_{S \to 0} S.A_o\left(\overset{*}{s}\right) = N_2(0) / D_2^{'}(0) = N_2 / D_2 \tag{138}$$

Where

$$N_2 = P_{70}\mu_o + \left(P_{01}P_{70} + P_{78}P_{81}\right)\mu_1 + \mu_2 + P_{23}\mu_3 + P_{24}\mu_4 + \left(P_{23}P_{36} + P_{24}P_{46}\right)\mu_6 + \mu_7 + P_{78}\mu_8 \tag{139}$$

And

$$D_2 = P_{70}\mu_o + \left(P_{78}P_{81} + P_{01}P_{70}\right)\mu_1 + m_1 + P_{23}\mu_3 + P_{24}m_2 + \left(P_{23}P_{36} + P_{24}P_{46}\right)\mu_6 + m_3 + P_{78}m_4 \tag{140}$$

8. Busy Period Analysis

Defining $B_1(t)$ = P [system initially started from regenerative state S_1 is under repair at epoch t]
We have the following recursive relations among B_1 (t)

$$\beta_o(t) = q_{01}(t)\ell\beta_1(t) + q_{02}(t)\ell\beta_2(t)$$
$$\beta_1(t) = q_{12}(t)\ell\beta_2(t)$$
$$\beta_2(t) = W_2(t) + q_{23}(t)\ell\beta_3(t) + q_{27}^{(5)}(t)\ell\beta_7(t)$$
$$\beta_3(t) = q_{36}(t)\ell\beta_6(t) + q_{37}(t)\ell\beta_7(t)$$
$$\beta_4(t) = W_4(t) + q_{46}(t) + \ell\beta_6(t) + q_{47}^{(5)}(t)\ell\beta_7(t)$$
$$\beta_6(t) = q_{67}(t)\ell\beta_7(t)$$
$$\beta_7(t) = W_7(t) + q_{70}(t)\ell\beta_o(t) + q_{78}(t)\ell\beta_5(t) + q_{72}^{(a)}(t)\ell\beta_2(t)$$
$$\beta_8(t) = W_8(t) + q_{81}(t) + \ell\beta_1(t) + q_{82}^{(a)}(t)\ell\beta_2(t) \tag{141-148}$$

Where

$$W_2^*(o) = K_1, W_4^*(o) = K_2, W_7^*(o) = K_3 \; and \; W_8^*(o) = K_4 \tag{149}$$

Taking laplace transform of relation (141-148), we get a set of linear equations in β_1^* (s) whose solution is obtained by omitting the argument "s" fir breuity may be expressed in the matrix form as

$$\left(\beta_o^*, \beta_1^*, \beta_2^*, \beta_3^*, \beta_4^*, \beta_6^*, \beta_7^*, \beta_8^*,\right) = Q^{-1}\left(o, o, W_2^*, o, W_4^*, o, W_7^*, W_8^*\right) \tag{150}$$

Where

$$Q = \begin{pmatrix} 1 & -q_{01} & -q_{02} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -q_{12}^{*} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{23}^{*} & -q_{24}^{*} & 0 & -q_{27}^{*(5)} & 0 \\ 0 & 0 & 0 & 1 & 0 & -q_{36}^{*} & -q_{37}^{*} & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{46}^{*} & -q_{47}^{*(5)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -q_{67}^{*} & 0 \\ -q_{70}^{*} & 0 & -q_{72}^{*(a)} & 0 & 0 & 0 & 1 & -q_{78}^{*} \\ 0 & -q_{81}^{*} & -q_{82}^{*(a)} & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Computing the relevant cofactors of first column elements of Q, the solution of (150) gives the laplace transform of $B_o(t)$ as $\beta_o^*(t) = N_3(s)/D_2(s)$

When D_2 (s) is same in (137)

And

$$N_3(s) = \left(q_{01}^{*}q_{12}^{*} + q_{02}^{*}\right)\left[W_2^{*} + q_{24}^{*}W_4^{*} + \left(W_7^{*} + q_{78}^{*}W_8^{*}\right)\left(q_{23}^{*}q_{36}^{*}q_{67}^{*} + q_{23}^{*}q_{27}^{*(5)} + q_2^{*} + q_{46}^{*}q_{67}^{*} + q_{24}^{*}q_{47}^{*(5)}\right)\right] \quad (151)$$

In the long run the fraction of time for which the system is under repair is given by

$$\beta_o = \lim_{t\to} \beta_o(t) = \lim s.\beta_o^*(s) = N_3 / D_2 \quad (152)$$

Where D_2 is same as (140) and

$$N_3 = \left[K_1 + P_{24}K_2 + K_3 + P_{72}K_4\right] \quad (153)$$

9. Expected Number of Visits by the Repairman

Using the earlier definition of V_1 (t) the following recursive relations among V_1 (t) are:

$$V_o(t) = Q_{01}(t)\$V_1(t) + Q_{02}(t)\$[1+V_2(t)]$$
$$V_1(t) = Q_{12}(t)\$[1+V_2(t)]$$
$$V_2(t) = Q_{23}(t)\$V_3(t) + Q_{24}(t)\$V_4(t) + Q_{27}^{(5)}(t)\$V_7(t)$$
$$V_3(t) = Q_{37}(t)\$[1+V_7(t)] + Q_{36}(t)\$V_6(t)$$
$$V_4(t) = Q_{46}(t)\$V_6(t) + Q_{47}^{(5)}(t)\$V_7(t)$$
$$V_6(t) = Q_{67}(t)\$[1+V_7(t)]$$
$$V_7(t) = Q_{70}(t)\$V_o(t) + Q_{78}(t)\$V_8(t) + Q_{72}^{(a)}(t)\$V_8(t)$$
$$V_8(t) = Q_{81}(t)\$V_1(t) + Q_{82}^{(a)}(t)\$V_2(t)$$

(154-161)

Taking Laplace stieltjes transform of these relations and solving for $\tilde{V}_o(s)$ after omitting the arguments "s" for brevity, we get

$$\tilde{V}_o(s) = N_4(s)/D_3(s) \qquad (162)$$

Where

$$N_4(s) = \left(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\right)\left[1 + \{\tilde{Q}_{23}\left(\tilde{Q}_{37} + \tilde{Q}_{36}\tilde{Q}_{37}\right) + \tilde{Q}_{24}\tilde{Q}_{46}\tilde{Q}_{67}\right]\{1 - \left(\tilde{Q}_{78}\tilde{Q}_{82}^{(a)} + \tilde{Q}_{72}^{(a)}\right) - \left(\tilde{Q}_{24}\tilde{Q}_{57}^{(5)} + \tilde{Q}_{27}^{(5)}\right)$$
$$\tilde{Q}_{72}^{(a)} + \tilde{Q}_{78}\tilde{Q}_{82}^{(a)} \qquad (163)$$

And

$$D_3(s) = 1 - \left[\tilde{Q}_{23}\left(\tilde{Q}_{37} + \tilde{Q}_{36}\tilde{Q}_{67}\right) + \tilde{Q}_{24}\left(\tilde{Q}_{46}\tilde{Q}_{67} + \tilde{Q}_{47}^{(5)}\right)\tilde{Q}_{27}^{(5)}\right]$$
$$\left[\tilde{Q}_{78}\left(\tilde{Q}_{81}\tilde{Q}_{12} + \tilde{Q}_{82}^{(a)}\right) + \tilde{Q}_{72}^{(a)} + \left(\tilde{Q}_{01}\tilde{Q}_{12} + \tilde{Q}_{02}\right)\tilde{Q}_{70}\right]$$

(164)

$$V_o = \lim_{t \to \infty} \frac{[V_o(t)]}{t} = \lim_{s \to o} s.\tilde{V}_o(s) = N_4 / D_2 \qquad (165)$$

Where D_2 is same as (140) and

$N_3 = P_{70} + P_{23} + P_{24}P_{46} + P_{78}P_{81}$ (166)

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