

ANALYSIS OF A SINGLE UNIT SHOCK MODEL BY USING REGENERATIVE POINT GRAPHICAL TECHNIQUE

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Abstract

For the doing the analysis of a stochastic system rapidly, the key reliability characteristics should be easily and quickly evaluated. Gupta [4] introduced a new technique called *Regenerative Point Graphical Technique* (RPGT) for doing the reliability analysis of a stochastic system by finding quickly and easily all the key reliability characteristics and other parameters of the system like mean time to system failure, availability of the system, busy period of the server, the number of visits of the server and the number of replacements in the long run of the system (under steady state conditions). This paper presents the analysis of two models of a single unit system with different types of repair policies of a single server that appears and disappears from the system randomly and where the system undergoes a random shock. The system may be working in a partially failed state after the impact of the shock and the analysis of the system is done by using RPGT.

Keywords: Shock, Impact, Availability, Regenerative State, RPGT, Fuzziness Measure.

1. Introduction:

The researchers including Chander & Bansal [1], Chander [2] and Malik et al [8] have used the Regenerative Point Technique (RPT) for the analysis of the various stochastic systems. They solved the state equations recursively after taking the necessary transforms, to determine the parameters of the stochastic systems (under steady state conditions). The complexity for the evaluation of the key parameters of the system increases with the increase in the number of states, as it becomes very time consuming and cumbersome to write all the state equations and then solve the transformed state equations and then taking the consequent limits using the particular formula of the RPT, along with a lot of simplifications, because of the complexity of the transition diagram of the stochastic system.

Gupta et al [3,5-7] have done the analysis of various systems by using RPGT for determining the mean time to system failure (MTSF), availability of the system, busy period of the server, number of visits of the server etc.(under the steady state conditions).

Chander [2] studied two models of a single unit system with different types of repair policies of a single server that appears and disappears from the system randomly and the system can undergo a random shock. Whenever a shock occurs, then the system may remain in the same state without any effect, with some probability or it may transit to a state with its failure rate

increased to the maximum level and further the system can fail completely there after. The system can fail completely during its normal operation. The author has considered that the system is similarly available in each of the working states before and after the impact of the shock, although its failure rate is increased to the highest level due to the impact of the shock. But, since the efficiency of the system may decrease after the impact of the shock and the availability is discounted which will result into the loss of the revenue. Therefore, the system ought to be treated in the partially failed (degenerated) state after the impact of the shock in such a situation. The possible applications of such a system are computer, generator, motor, a fly-over bridge etc. The *objective* of this paper is to determine the key statistical parameters of the system (under steady state conditions) by using RPGT and by treating the state of the system after the impact of the shock as a partially failed state by discounting the mean sojourn times of the said available states by using the fuzziness measure of the state.

2. The System:

The system is a single unit repairable system with random appearance and disappearance of the repairing facility and the repair is admissible in two Phases -1 & 2. In Model I, the server repairs the partially failed system (Phase-1) immediately after the impact of the shock and also on its complete failure (Phase-2), while in Model II, the server repairs the system only on its complete failure (Phase-2). The p.d.f. of the failure times of the system are exponential where that of the shock time & repair time are general distribution functions. The p.d.f. of the server's availability or non-availability at the system, are exponential with constant rates of appearance/disappearance of the server. The transition diagrams of the Model I and Model II are as shown in Fig.1 & Fig. 2 respectively.

2.1 Assumptions & Notations:

The following assumptions and notations are used in the analysis of the system:

- 1) The system starts from a good state at time $t = 0$.
- 2) The system may fail completely due to its normal operation or may fail partially due to the impact of the shock before its complete failure.
- 3) There is single repair facility and the server appears and disappears randomly from the system. The repairs can

start only if the server is available and further that the server can not leave the system while repairing it. After the repairs, the system is in good state.
 4) All random variables are independent and un-correlated.

$pr / P_{i,j}$: Probability/steady state transition probability from the regenerative state i to the regenerative state j , without visiting any other states.

$V_{0,i}$: Transition probability factor of the reachable state i from the 0-state.

$V_{k,k} / \overline{V_{k,k}}$: Transition probability factor of the reachable state k of the k -cycle/ k -cycle.

$(i \xrightarrow{sr} j)$: R -th directed simple path from i -state to j -state; r takes positive integral values for different paths from i -state to j -state.

$(0 \xrightarrow{sff} i)$: a directed simple failure free path from 0-state to i -state.

$(i, j) / (i, j, k)$: $(i, j) = P_{i,j}$; $(i, j, k) = (i, j) (j, k) = P_{i,j} \cdot P_{j,k}$

η_i : Expected time spent by the server while doing a job, given that the system entered the regenerative state ' i ' at $t = 0$.

μ_i / μ_i^1 : Mean sojourn time of the state ' i ' / total un-conditional time spent before transiting to any other regenerative state(s), given that the system entered regenerative state ' i ' at $t = 0$.

f_i : Fuzziness measure of the i -state; $f_i = 0$, if ' i ' is a failed state; $f_i = 1$, if ' i ' is an up state and $f_i \in (0, 1)$, if ' i ' is a partially failed state.

F_{ur} / F_{wr} : Unit is under repair/ waiting for repairs.

PF / PF_{ur} : Unit is partially failed/ partially failed

unit is under repairs.

(A)/(NA) : Server is available/not available at the system.

λ_0 / λ_1 : Constant failure rate of the unit before/after the impact of the shock.

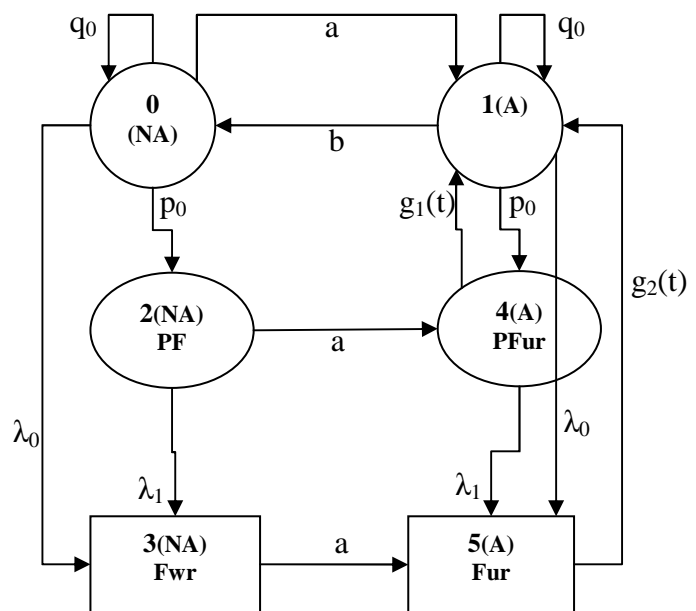
p_0/q_0 : Probability that the unit is affected/or not due to the shock.

a/b : Constant rate of appearance/disappearance of the server.

$h(t)/g_i(t)$: p.d.f. of the shock time/ p.d.f. of repair-time in Phase $i = 1, 2$.

State	Symbol
Regenerative State(s): 0 to 5	•
Up-State(s): 0,1	○
Partially Failed State(s): 2,4	◌
Failed State(s): 3,5	◻

Figure 1



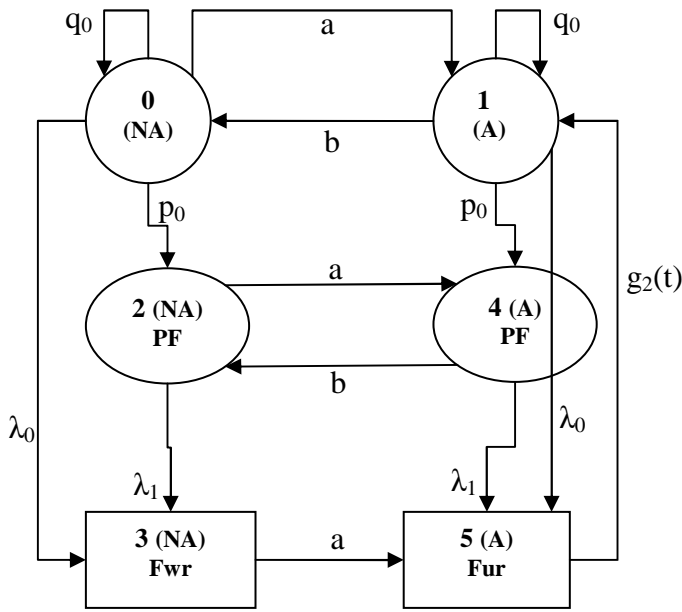


Fig.2

3. Evaluation of the Parameters of the System:

The mean time to system failure and the other key parameters of the system (under steady state conditions) are evaluated by using RPGT and '0' as the initial-state, as under:

3.1 Model I:

On writing,

$$U'_1 = (1,1) + (1,4,1); U'_2 = (1,1) + (1,5,1); U'_3 = (1,1) + (1,4,1) + (1,4,5,1) + (1,5,1);$$

$$U_1 = (1,1); U_4 = \frac{(4,1,4)}{1-U_1}; U_5 = \frac{(5,1,5)}{1-U_1}; U'_4 =$$

$$\frac{(4,1,4)}{1-U'_2} + \frac{(4,5,1,4)}{\{1-U_5\}\{1-U_1\}}; U'_5 = \frac{(5,1,4,5)}{1-U'_1} + \frac{(5,1,5)}{1-U'_1}$$

$$V_{0,0} = 1;$$

$$V_{0,1} = \frac{(0,1)}{1-U'_3} + \frac{(0,2,3,5,1) + (0,3,5,1)}{(1-U'_5)(1-U'_1)} + \frac{(0,2,4,1)}{(1-U'_4)(1-U'_2)} + \frac{(0,2,4,5,1)}{(1-U'_4)(1-U_5)(1-U_1)}$$

$$V_{0,2} = (0,2);$$

$$V_{0,3} = (0,2,3) + (0,3);$$

$$V_{0,4} = \frac{(0,1,4)}{1-U'_3} + \frac{(0,2,3,5,1,4) + (0,3,5,1,4)}{(1-U'_5)(1-U'_1)} + \frac{(0,2,4)}{1-U'_4}$$

$$V_{0,5} = \frac{(0,1,4,5) + (0,1,5)}{1-U'_3} + \frac{(0,2,3,5) + (0,3,5)}{1-U'_5} + \frac{(0,2,4,1,5)}{(1-U'_4)(1-U'_2)} + \frac{(0,2,4,5)}{(1-U'_4)(1-U_5)}$$

3.1a) MTSF:

From Fig.1, the regenerative un-failed states visited by the system before transiting to any failed states are: $i = 0,1,2,4$. The mean time to system failure is given by (using RPGT):

$$MTSF = \left[\sum_{i, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r(sff)} i)\} \cdot \mu_i}{\prod_{k_1 \neq 0} \{1 - V_{k_1, k_1}\}} \right\} \right] \div$$

$$\left[1 - \sum_{s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r(sff)} 0)\}}{\prod_{k_2 \neq 0} \{1 - V_{k_2, k_2}\}} \right\} \right] \dots (1)$$

$$T_1 = \left[1 \cdot \mu_0 + \left\{ \frac{(0,1)}{1-U'_1} + \frac{(0,2,4,1)}{(1-U_4)(1-U_1)} \right\} \mu_1 + (0,2) \mu_2 + \left\{ \frac{(0,1,4)}{1-U'_1} + \frac{(0,2,4)}{1-U_4} \right\} \mu_4 \right]$$

$$= \left[1 - (0,0) - \frac{(0,1,0)}{1-U'_1} - \frac{(0,2,4,1,0)}{U_4(1-U_1)} \right] =$$

$N_{11} \div D_{11}$ where

$$N_{11} = [(\mu_0 + p_{0,2} \mu_2)(1 - p_{1,1} - p_{1,4} p_{4,1}) + (p_{0,1} + p_{0,2} p_{2,4} p_{4,1}) \mu_1 + \{p_{0,1} p_{1,4} + p_{0,2} p_{2,4} (1 - p_{1,1})\} \mu_4]$$

$$D_{11} = [(1 - p_{0,0}) \cdot (1 - p_{1,1} - p_{1,4} p_{4,1}) -$$

$$p_{1,0} (p_{0,1} + p_{0,2} p_{2,4} p_{4,1})]$$

3.1b) Availability of the System:

From Fig.1 the regenerative available un-failed (including the partially failed) states are: $j = 0, 1, 2, 4$; $\mu'_j = \mu_j$. Therefore, total fraction of time for which the system is available is given by (using RPGT):

$$A_1 = \left[\sum_{j, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r} j)\} f_j \cdot \mu_j}{\prod_{k_1 \neq 0} \{1 - V_{k_1, k_1}\}} \right\} \right] \div \left[\sum_{i, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r} i)\} \cdot \mu_i^1}{\prod_{k_2 \neq 0} \{1 - V_{k_2, k_2}\}} \right\} \right]$$

$$A_1 = [\sum_j V_{0, j} \cdot f_j \cdot \mu_j] \div [\sum_i V_{0, i} \cdot \mu_i^1]$$

..... (2)

$$= [V_{0,0} f_0 \mu_0 + V_{0,1} f_1 \mu_1 + V_{0,2} f_2 \mu_2 + V_{0,4} f_4 \mu_4] \div D_{12} \text{ where}$$

$$D_{12} = [\sum_i V_{0, i} \cdot \mu_i^1] = [V_{0,0} \mu'_0 + V_{0,1} \mu'_1 + V_{0,2} \mu'_2 + V_{0,3} \mu'_3 + V_{0,4} \mu'_4 + V_{0,5} \mu'_5]$$

$$= [D_{12} / (1 - p_{0,0})] \text{ where}$$

$$D_{12} = [p_{1,0} \mu_0 + (1 - p_{0,0}) \mu_1 + p_{0,2} p_{1,0} \mu_2 + p_{1,0} (p_{0,3} + p_{0,2} p_{2,3}) \mu_3 + \{p_{1,4} (1 - p_{0,0}) + p_{1,0} p_{0,2} p_{2,4}\} \mu_4 + \{(1 - p_{0,0}) (p_{1,5} + p_{1,4} p_{4,5}) + p_{1,0} p_{0,2} (p_{2,3} + p_{2,4} p_{4,5}) + p_{1,0} p_{0,3}\} \mu_5]$$

$$A_1 = N_{12} \div D_{12} \text{ where } N_{12} = [p_{1,0} f_0 \mu_0 + (1 - p_{0,0}) f_1 \mu_1 + p_{0,2} p_{1,0} f_2 \mu_2 + \{p_{1,0} p_{0,2} p_{2,4} + p_{1,4} (1 - p_{0,0})\} f_4 \mu_4]$$

3.1c) Busy Period of the Server:

From Fig.1 the regenerative states where the server is busy: $j = 4, 5$ and $\eta_j = \mu_j$. Therefore, the total fraction of time for which the server remains busy is given by (using RPGT):

$$B_1 = \left[\sum_{j, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r} j)\} \eta_j}{\prod_{k_1 \neq 0} \{1 - V_{k_1, k_1}\}} \right\} \right] \div \left[\sum_{i, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r} i)\} \cdot \mu_i^1}{\prod_{k_2 \neq 0} \{1 - V_{k_2, k_2}\}} \right\} \right]$$

$$B_1 = [\sum_j V_{0, j} \cdot \eta_j] \div [\sum_i V_{0, i} \cdot \mu_i^1]$$

..... (3)

$$= [V_{0,4} \eta_4 + V_{0,5} \eta_5] \div [D_{12} / (1 - p_{0,0})] = N_{13} \div D_{12}$$

Where $N_{13} = [\{p_{1,4} (1 - p_{0,0}) + p_{1,0} p_{0,2} p_{2,4}\} \mu_4 + \{(1 - p_{0,0}) (p_{1,5} + p_{1,4} p_{4,5}) + p_{1,0} p_{0,2} (p_{2,3} + p_{2,4} p_{4,5}) + p_{1,0} p_{0,3}\} \mu_5]$

An D_{12} is already specified.

3.1d) Expected Number of Visits of the Server:

From fig. 1, the regenerative states where the server visits afresh along the different paths are: $j = 1, 4$ and 5 via the

states $x = 0, 2$ and 3 respectively. Therefore the number of visits of the server is given by (using RPGT):

$$V_1 = \left[\sum_{j, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r} j)\}}{\prod_{k_1 \neq 0} \{1 - V_{k_1, k_1}\}} \right\} \right] \div \left[\sum_{i, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r} i)\} \cdot \mu_i^1}{\prod_{k_2 \neq 0} \{1 - V_{k_2, k_2}\}} \right\} \right]$$

$$V_1 = \left[\sum_{x, j} V_{0, x}(x, j) \right] \div \left[\sum_i V_{0, i} \cdot \mu_i^1 \right]$$

....(4)

$$= [V_{0,0}(0,1) + V_{0,2}(2,4) + V_{0,3}(3,5)]$$

$$\div [D_{12} / (1 - p_{0,0})]$$

$$= N_{14} \div D_{12} \text{ where } N_{14} = (1 - p_{0,0}) p_{1,0} \text{ and } D_{12} \text{ is already specified.}$$

3.2 Model II:

On writing: $L_1 = (1,1)$, $L_2 = (2,4,2)$; $L_4 = (4,2,4)$;

$$L_5 = \frac{(5,1,5)}{1 - L_1}; L_4' = \frac{(4,5,1,4)}{\{1 - L_5\} \{1 - L_1\}};$$

$$L_{1,4} = \{1 - L_1\} \{1 - L_4\}; L_5' = \frac{(5,1,4,5)}{1 - L_1} + \frac{(5,1,5)}{1 - L_1}; L_5''$$

$$= \frac{(5,1,4,5)}{\{1 - L_1\} \{1 - L_4\}} + \frac{(5,1,5)}{1 - L_1}$$

$$L_1' = (1,1) + \frac{(1,4,2,3,5,1)}{1 - L_4} + \frac{(1,4,5,1)}{1 - L_4} + (1,5,1); L_2'$$

$$= \frac{(2,3,5,1,4,2)}{\{1 - L_5'\} \{1 - L_1\}} + \frac{(2,4,2)}{1 - L_4};$$

$$L_3' = \frac{(3,5,1,4,2,3)}{\{1 - L_5''\} \{1 - L_1\} \{1 - L_4\}}; L_{1,4}' = (1 - L_1')(1 - L_4)$$

$$; L_{2,4} = (1 - L_2')(1 - L_4) \quad L_{2,5} = (1 - L_2')(1 - L_5');$$

$$L_{2,5,1} = (1 - L_2')(1 - L_5')(1 - L_1);$$

$$L_{2,4,5,1} = (1 - L_2')(1 - L_4')(1 - L_5')(1 - L_1);$$

$$L_{3,5,1} = (1 - L_3')(1 - L_5'')(1 - L_1);$$

$$L_{3,5,1,4} = (1 - L_3')(1 - L_5'')(1 - L_1)(1 - L_4).$$

$$V_{0,0} = 1;$$

$$V_{0,1} = \frac{(0,1)}{1 - L_1'} + \frac{(0,2,3,5,1)}{L_{2,5,1}} + \frac{(0,2,4,5,1)}{L_{2,4,5,1}} + \frac{(0,3,5,1)}{L_{3,5,1}}$$

$$V_{0,2} = \frac{(0,1,4,2)}{L_{1,4}'} + \frac{(0,2)}{1 - L_2'} + \frac{(0,3,5,1,4,2)}{L_{3,5,1,4}};$$

$$V_{0,3} = \frac{(0,1,4,2,3)}{L_{1,4}'} + \frac{(0,2,3)}{1 - L_2'} + \frac{(0,3)}{1 - L_3'}$$

$$V_{0,4} = \frac{(0,1,4)}{L_{1,4}'} + \frac{(0,2,3,5,1,4)}{L_{2,5,1}} + \frac{(0,2,4)}{L_{2,4}} + \frac{(0,3,5,1,4)}{L_{3,5,1,4}}$$

$$V_{0,5} = \frac{(0,1,4,2,3,5) + (0,1,4,5)}{L_{1,4}'} + \frac{(0,1,5)}{1 - L_1'} + \frac{(0,2,3,5)}{L_{2,5}}$$

$$+ \frac{(0,2,4,5)}{L_{2,4,5}} + \frac{(0,3,5)}{(1 - L_3')(1 - L_5'')}$$

3.2a) MTSF:

From Fig.2, the regenerative un-failed states visited by the system before transiting to any failed states are: $i = 0, 1, 2, 4$. The mean time to system failure is given by (1) as under:

$$T_2 =$$

$$\left[1 \cdot \mu_0 + \frac{(0,1)}{1 - L_1} \mu_1 + \left\{ \frac{(0,1,4,2)}{L_{1,4}'} + \frac{(0,2)}{1 - L_2'} \right\} \mu_2 \right]$$

$$+ \left\{ \frac{(0,1,4)}{L_{1,4}'} + \frac{(0,2,4)}{1 - L_2'} \right\} \mu_4$$

$$= N_{21} \div D_{21} \text{ where } N_{21} = \{ (1 - p_{1,1}) \mu_0 + p_{0,1} \mu_1 \} \{ 1 - p_{2,4} p_{4,2} \} + \{ p_{0,2} (1 - p_{1,1})$$

$$+ p_{0,1} p_{1,4} p_{4,2} \} \mu_2 + \{ p_{0,1} p_{1,4} + p_{0,2} p_{2,4} (1 - p_{1,1}) \} \mu_4] \text{ and}$$

$$D_{21} = [\{ (1 - p_{0,0}) \cdot (1 - p_{1,1}) - p_{0,1} p_{1,0} \} \{ 1 - p_{2,4} p_{4,2} \}]$$

3.2b) Availability of the System:

From Fig.2, the regenerative states at which the system is available are: $j = 0, 1, 2, 4$; $\mu'_j = \mu_j$. Therefore, total fraction of time for which the system is available is obtained by using (2) as under: $A_2 = \sum_j V_{0,j} \cdot f_j \cdot \mu_j \div [\sum_i V_{0,i} \cdot \mu_i^1]$

$$A_2 =$$

$$[V_{0,0} f_0 \mu_0 + V_{0,1} f_1 \mu_1 + V_{0,2} f_2 \mu_2 + V_{0,4} f_4 \mu_4] \div [V_{0,0} \mu'_0 + V_{0,1} \mu'_1 + V_{0,2} \mu'_2 + V_{0,3} \mu'_3 + V_{0,4} \mu'_4 + V_{0,5} \mu'_5]$$

$$= N_{22} \div D_{22} \text{ where}$$

$$N_{22} = [\{ p_{1,0} f_0 \mu_0 + (1 - p_{0,0}) f_1 \mu_1 \} \{ 1 - p_{2,4} p_{4,2} \} + \{ p_{0,2} p_{1,0} + (1 - p_{0,0}) p_{1,4} p_{4,2} \} f_2 \mu_2 + \{ p_{1,0} p_{0,2} p_{2,4} + p_{1,4} (1 - p_{0,0}) \} f_4 \mu_4] \text{ and}$$

$$D_{22} = [\{ p_{1,0} \mu_0 + (1 - p_{0,0}) \mu_1 \} \{ 1 - p_{2,4} p_{4,2} \} + \{ p_{1,0} p_{0,2} + (1 - p_{0,0}) p_{1,4} p_{4,2} \} \mu_2 + \{ (1 - p_{0,0}) p_{1,4} p_{4,2} p_{2,3} + p_{1,0} p_{0,3} (1 - p_{2,4} p_{4,2}) + p_{1,0} p_{0,2} p_{2,3} \} \mu_3 + \{ p_{1,0} p_{0,2} p_{2,4} + p_{1,4} (1 - p_{0,0}) \} \mu_4$$

$$+ \{ \{ (1 - p_{0,0}) \cdot (1 - p_{1,1}) - p_{1,0} p_{0,1} \} \{ 1 - p_{2,4} p_{4,2} \} \} \mu_5]$$

3.2c) Busy Period Analysis of the Server:

From Fig.2, the regenerative state where the server is busy doing repairs is $j = 5$; $\mu_i^1 = \mu_i \forall i = 0$ to 5 and $\eta_5 = \mu_5$. Therefore, the busy time of the server, doing repairs is obtained by using (3) as under:

$$B_2 = V_{0,5} \eta_5 \div \{ D_{22} / \{ p_{1,0} (1 - p_{2,4} p_{4,2}) \} \} = N_{23} \div D_{22} \text{ where}$$

$$N_{23} = [\{ (1 - p_{0,0}) \cdot (1 - p_{1,1}) - p_{1,0} p_{0,1} \} (1 - p_{2,4} p_{4,2})] \mu_5 \text{ and } D_{22} \text{ is already specified.}$$

3.2d) Expected Number of Visits of the Server:

From fig. 2, the regenerative states where the server visits afresh along the different paths are: $j = 1, 4$ and 5 via the states $x = 0, 2$ and 3 respectively. Therefore the number of visits of the server is given by using (4) as under:

$$V_2 = [V_{0,0} (0,1) + V_{0,2} (2,4) + V_{0,3} (3,5)] \div [$$

$$D_{22} / \{ p_{1,0} (1 - p_{2,4} p_{4,2}) \}] = N_{24} \div D_{22}$$

Where $N_{24} = [(1 - p_{0,0}) (p_{1,0} + p_{1,4} p_{4,2}) - p_{2,4} p_{4,2} (p_{0,3} + p_{0,1}) p_{1,0}]$ and D_{22} is already specified.

3.3 Special Case:

On taking $f_j = 1$ for all the available states including the partially reduced states, the above results reduce to the results as are obtained by Chander [2]. The above results which are obtained more quickly and easily by using RPGT, in comparison to that are obtained by using RPT can be used for doing further analysis of the system more rapidly.

4. Conclusion:

The profit analysis can be done by using the function of the system is: $P_i = C_1 \cdot A_i - C_2 \cdot B_i - C_3 \cdot V_i$ Where $i = 1$ for Model I and $i = 2$ for Model II and

$$C_1 = \text{Revenue per unit of time the system is}$$

available.

C_2 = Cost per unit time
 the server remains
 busy for the repairs.

C_3 = Cost per visit of the
 server.

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